

THE DIVERSIFIABILITY OF EXCHANGE RISK

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Discussions of the determinants of the spot or forward exchange rate frequently argue that holders of foreign assets that are subject to exchange risk must be compensated by a risk premium. This paper shows that much of exchange risk is diversifiable. If there are no outside assets and the value of the currency is uncorrelated with the value of other forms of wealth, then *all* exchange risk is diversifiable; there is no risk premium. More generally, there is a risk premium, but it need not be related to foreign indebtedness or to the variability of the exchange rate as commonly presumed.

1. Introduction

It is often argued that if the future spot exchange rate is uncertain and if market participants are risk-averse, then the forward rate will differ from the expected future spot rate by a risk premium.¹ Market participants will be prepared to buy forward exchange only up to the point where the expected return is cancelled out by the marginal risk of their open position. Because of the existence of exchange risk, the demand for foreign exchange will be less than perfectly elastic with respect to the forward rate, and there will be no reason for the clearing of the forward market necessarily to take place at the point where the forward rate is equal to the expected future spot rate.

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¹The argument has important implications in two areas: (1) tests of efficiency in the forward exchange market, and (2) models of spot rate determination. (1) The implication for tests of efficiency is that the expected future spot rate can differ from the forward rate, even though the market is efficient, if the domestic country is a net creditor or net debtor. Levich (1979) surveys tests of efficiency in the forward exchange market, and concludes that they are not valid since they test efficiency jointly with the hypothesis of a zero risk premium. (2) The conclusions regarding the forward rate carry through to the spot rate (by covered interest parity), given the interest differential and expected future spot rate. The effect of an increase in the quantity of foreign debt is to cause the exchange rate to fall. This effect is appealing because it says that the price of foreign assets (the exchange rate) falls when their supply increases, just as with any good that faces less-than-perfectly-elastic demand. The effect of an increase in variability is described by Schadler (1977, p. 286): 'If the domestic country is a net debtor (creditor), the increase in uncertainty will force the exchange rate to depreciate by more (less) or appreciate by less (more) than if the degree of uncertainty had not changed.'

Furthermore, it is conventionally presumed that an increase in domestic holdings of foreign debt will increase exposure to risk and cause a downward shift in the demand schedule for forward exchange and thus a fall in the forward exchange rate. And it is conventionally presumed that if the domestic country is a net creditor, an increase in the variability of the future value of the foreign currency, with no change in its expectation, will also increase exposure to risk and cause a fall in the forward exchange rate. (If the domestic country is a net debtor, the presumption is reversed.)

It is the contention of this paper that much of the literature on exchange risk has ignored the fact that when domestic residents hold claims on foreign residents, such as bonds or forward exchange contracts, *variability in the exchange rate creates risk not only for domestic residents, but also for foreign residents.*² The two risks are offsetting: an unexpected rise in the exchange rate raises the value of domestic residents' assets *and* the cost of foreign residents' liabilities. Residents of the two countries can trade these risks with each other, either by agreeing to denominate debt in a weighted average of the two currencies or (if institutional factors dictate that foreign debt be denominated in foreign currency) by the domestic creditors selling foreign exchange to foreign debtors on the forward market, allowing both sides to hedge their exchange risks. In the resulting market equilibrium, *all exchange risk can be diversified away*, under certain conditions. Then the risk premium will be equal to zero despite the presence of uncertainty and risk-aversion. Under more general conditions, a nonzero risk premium does indeed separate the forward rate from the expected future spot rate; but the risk premium is still not necessarily related to foreign indebtedness and variability of the exchange rate in the way conventionally presumed.

The paper demonstrates the following three propositions in a one-good model with no transactions costs:

(i) There is no risk premium if two conditions are met. (i) All assets are nominal assets (money and bonds) whose returns are fixed in terms of existing currencies, or else – if there are other assets (goods and human and nonhuman capital) – their real returns are independent of currency values. The latter condition obtains, for example, in exchange rate models in which real output is assumed exogenously determined in the short run.³ (ii) All nominal assets are 'inside' assets, that is, either they are privately-issued debt

²The reference is to the so-called 'modern' theory of forward exchange: Tsiang (1959), Feldstein (1968), Grubel (1968) and Leland (1971). This literature ignores the existence of foreign residents, in that the market is assumed to evaluate wealth solely in domestic terms. Thus the ownership of foreign assets carries exchange risk that must be compensated by a risk premium, but the ownership of domestic assets does not. The more recent finance models do include foreign residents in the market for forward exchange. See Solnik (1974), Grauer, Litzenberger and Stehle (1976), Kouri (1976a, 1976b) and Fama and Farber (1979). Some of the conclusions in the present paper are implicit in this literature.

³See, for example, Dornbusch (1976) and Frankel (1979).

or else – if they include assets issued by the government – they are viewed by the residents of the country as entailing offsetting liabilities such as future taxation.

(2) If there are ‘outside’ assets that are assumed denominated in the currency of the country that issues them, then there is a risk premium. The risk premium is positively (negatively) related to the stock of foreign (domestic) assets, holding total wealth constant. The risk premium is also positively (negatively) related to the variability of the value of the foreign (domestic) currency to the extent that the outside assets are issued by the domestic (foreign) government. This conclusion is similar to the common presumption, except that what matters is the government’s domestically-denominated indebtedness, whether to its own citizens or to foreigners, not the net indebtedness of residents of one country to residents of the other.

(3) If the value of a country’s currency is correlated with the value of real assets, then there is again a risk premium. But the presumption that the risk premium is positively related to the variability in the value of the foreign currency holds only if real disturbances come from the supply side (for example, variations in the weather or the terms of trade) so that the country’s price level is negatively correlated with its real output. If real disturbances come from the demand side (for example, variations in fiscal or monetary policy) so that the country’s price level is positively correlated with its real output, then the presumption is reversed.

2. The model

The model used here is the simplest one possible that is capable of answering the relevant questions. In each period, individuals allocate their financial wealth between B units of domestically-denominated assets, in terms of their known value in domestic currency next period, and B^* units of foreign-denominated assets, in terms of their known value in foreign currency next period. S_{+1} is defined as next period’s spot exchange rate (the price of foreign currency in terms of domestic currency), which is uncertain this period. F is defined as the *current* price of foreign assets, payable next period, in terms of domestic assets.⁴ We are going to find the value of F that clears the market for assets, and see how it is affected by variability in currency values and by net domestic or foreign indebtedness, relative to the expected future spot rate.

⁴If the domestic and foreign assets are bonds that pay interest each period of r and r^* , respectively, then the price of assets (payable next period) in terms of this period’s currency is $1/(1+r)$ and $1/(1+r^*)$, respectively. If S is the current price of foreign currency (payable on the spot) in terms of domestic currency, then covered interest parity says that the current price of foreign assets (payable next period) in terms of domestic assets, F , is equal to $S(1+r)/(1+r^*)$.

The natural interpretation of F is that it is the forward exchange rate.⁵ Without changing anything in the analysis that follows, we could give a simpler interpretation to the model by assuming that non-interest-bearing money is the only asset. F would be identical to the current spot rate. Expected future appreciation or depreciation would then in itself constitute a risk premium. Though all the insights of the model could be obtained under this simpler interpretation, we will maintain the more realistic interpretation that B and B^* include interest-paying bonds and that F is the forward exchange rate.

The next-period value of all real assets (human and nonhuman capital) is represented by X_{+1} in real terms. π_{+1} and π_{+1}^* are defined as the inverse of the price level (the real value of the currency) next period, at home and abroad, respectively. There is only one good in the model (or, equivalently, one basket of goods), so purchasing power parity always holds: $\pi_{+1}S = \pi_{+1}^*$. X , π_{+1} , π_{+1}^* and S_{+1} are all uncertain this period. No presumption is made as to direction of causality among them.

The final assumption is that individuals allocate their financial wealth so as to maximize the expected value of a utility function which is quadratic in the real value of next period's wealth. Intertemporal allocations play no role. Some people may be borrowers and some lenders, for whatever reasons; all that matters is how they allocate their (positive or negative) financial wealth.

Individual i 's next period wealth is

$$W_{i,+1} = B_i + S_{+1}B_i^* + \frac{1}{\pi_{+1}} X_i$$

in domestic nominal terms, or $\pi_{+1}W_{i,+1} = \pi_{+1}(B_i + S_{+1}B_i^*) + X_{i,+1}$ in real terms. He/she acts to maximize the expected value of

$$\begin{aligned} U_{i,+1} &\equiv \pi_{+1}W_{i,+1} - b_i(\pi_{+1}W_{i,+1})^2 \\ &= \pi_{+1}(B_i + S_{+1}B_i^*) + X_{i,+1} - b_i[\pi_{+1}(B_i + S_{+1}B_i^*) + X_{i,+1}]^2, \end{aligned}$$

where b_i is a measure of risk-aversion.⁶ Letting E represent the expectation

⁵The applicability to the question of tests of the efficiency of the forward market is direct. The applicability to the question of the determination of the spot rate requires the assumption that interest rates are held constant. For this purpose, we can imagine that any changes in the stock of assets are made in such a way as to keep interest rates unchanged. If one believes in a portfolio-balance explanation of the demand for money, this means assuming that the stocks of bonds and money are always changed equiproportionately. If one believes in a transactions explanation of the demand for money, this means assuming that only the stock of bonds is changed.

⁶The quadratic utility function can only be used over the range $(\pi W) < (\frac{1}{2}b)$, since the marginal utility for real wealth becomes negative after that point. However, the results can easily be generalized to the case where utility is any function of the mean and variance of real wealth

operator, the quantity to be maximized is EU_{i+1} . Since the current price of foreign assets in terms of domestic assets is F , the condition for maximization is that F equals the marginal rate of substitution between foreign and domestic assets:

$$F = \frac{\partial EU_{i+1} / \partial B_i^*}{\partial EU_{i+1} / \partial B_i} = \frac{E(\pi_{+1} S_{+1}) - 2b_i E\pi_{+1} S_{+1} [\pi_{+1} (B_i + S_{+1} B_i^*) + X_{i+1}]}{E(\pi_{+1}) - 2b_i E\pi_{+1} [\pi_{+1} (B_i + S_{+1} B_i^*) + X_{i+1}]}, \tag{1}$$

where the derivative operator has been passed through the expectation operator. We multiply through by the denominator of the last expression and remember that B_i and B_i^* are not random variables:

$$\begin{aligned} F & \left[\frac{1}{2b_i} E\pi_{+1} - (B_i E\pi_{+1}^2 + B_i^* E\pi_{+1}^2 S_{+1} + E\pi_{+1} X_{i+1}) \right] \\ & = \frac{1}{2b_i} E\pi_{+1} S_{+1} - (B_i E\pi_{+1}^2 S_{+1} + B_i^* E\pi_{+1}^2 S_{+1}^2 + E\pi_{+1} X_{i+1} S_{+1}). \end{aligned}$$

Now we sum the equation over all individuals i :

$$\begin{aligned} F & \left[\frac{1}{b} E\pi_{+1} - (\bar{B} E\pi_{+1}^2 + \bar{B}^* E\pi_{+1}^2 S_{+1} + E\pi_{+1} X_{+1}) \right] \\ & = \frac{1}{b} E\pi_{+1} S_{+1} - (\bar{B} E\pi_{+1}^2 S_{+1} + \bar{B}^* E\pi_{+1}^2 S_{+1} + E\pi_{+1} X_{+1} S_{+1}). \end{aligned}$$

where we have made the following definitions:

$$\begin{aligned} \frac{1}{b} & \equiv \sum_i \frac{1}{2b_i} \quad (\text{a measure of aggregate risk-aversion}), \\ \bar{B} & \equiv \sum_i B_i \quad (\text{the stock of domestically denominated outside assets, since all inside assets cancel out}), \\ \bar{B}^* & \equiv \sum_i B_i^* \quad (\text{the stock of foreign-denominated outside assets}) \end{aligned}$$

(whereas in the quadratic case, utility is a *linear* function of the mean and variance). Under the axioms of expected utility maximization, the underlying stochastic process must have a normal distribution if the function to be maximized is to be an arbitrary function of the mean and variance. To generalize the results, it is sufficient to interpret b_i as $-(\partial U_i / \partial \text{mean}) / (\partial U_i / \partial \text{var})$, which is not in general constant. I am indebted to Hal Varian for this point.

and

$$X \equiv \sum_i X_i \quad (\text{the aggregate value of real assets}).$$

We solve for the current exchange rate F :

$$F = \frac{E\pi_{+1}S_{+1} - b(E\pi_{+1}X_{+1}S_{+1} + \bar{B}E\pi_{+1}^2S_{+1} + \bar{B}^*E\pi_{+1}^2S_{+1}^2)}{E\pi_{+1} - b(E\pi_{+1}X_{+1} + \bar{B}E\pi_{+1}^2 + \bar{B}^*E\pi_{+1}^2S_{+1})}$$

Finally, we use the purchasing power parity condition $\pi_{+1}S_{+1} = \pi_{+1}^*$:

$$F = \frac{E\pi^* - b(E\pi^*X + \bar{B}E\pi^* - \bar{B}^*E\pi^{*2})}{E\pi - b(E\pi X + \bar{B}E\pi^2 + \bar{B}^*E\pi\pi^*)} \quad (2)$$

This is the basic equation for the current forward exchange rate which is used throughout the remainder of this paper. Notice that the formula does not depend on any characteristics of individuals. In particular it does not matter if domestic residents are indebted to foreign residents or if they are more risk-averse than foreign residents. Only aggregate variables matter: in particular the stock of domestically-denominated outside assets, the stock of foreign-denominated outside assets, the aggregate value of real assets, and aggregate risk-aversion.

This derivation can be made somewhat more intuitive by observing that the marginal expected utility of an additional asset unit is given by its marginal expected real value (the inverse of the price level) minus marginal exposure to real risk (the covariance of the real value of the asset with the real value of wealth). Thus expression (1), representing the ratio of the marginal expected utilities, can be interpreted as:

$$F = \frac{\text{expected} \left(\begin{array}{c} \text{(future value} \\ \text{of foreign} \\ \text{currency)} \end{array} \right) - \left(\begin{array}{c} \text{(a measure} \\ \text{of } i\text{'s risk-} \\ \text{aversion)} \end{array} \right) \text{expected} \left(\left(\begin{array}{c} \text{(future value} \\ \text{of foreign} \\ \text{currency)} \end{array} \right) \left(\begin{array}{c} \text{(future} \\ \text{value of} \\ \text{ } i\text{'s wealth)} \end{array} \right) \right)}{\text{expected} \left(\begin{array}{c} \text{(future value} \\ \text{of domestic} \\ \text{currency)} \end{array} \right) - \left(\begin{array}{c} \text{(a measure} \\ \text{of } i\text{'s risk-} \\ \text{aversion)} \end{array} \right) \text{expected} \left(\left(\begin{array}{c} \text{(future value} \\ \text{of domestic} \\ \text{currency)} \end{array} \right) \left(\begin{array}{c} \text{(future} \\ \text{value of} \\ \text{ } i\text{'s wealth)} \end{array} \right) \right)}$$

where the future value of i 's wealth depends on the asset holdings he chooses this period and the future value of the two currencies. The manipulations after expression (1) show that the equation can be aggregated over all individuals. Eq. (2) looks similar to eq. (1) and has essentially the same intuitive explanation, except that individual i 's measure of risk-aversion and future wealth are replaced by a measure of *aggregate* risk-aversion and future *aggregate* wealth, respectively

⁷Henceforth the time subscript (+1) is omitted.

3. When there is no risk premium

First, consider the case of risk-neutrality, where $b=0$. (It is sufficient that $b_i=0$ for any single individual – assuming, of course, unlimited access to capital.)

$$F = E\pi^*/E\pi.$$

The current exchange rate is given by the ratio of the expected values of the purchasing powers of the two currencies. In the special case in which the value of domestic currency π is a constant, $F = E\pi S/\pi = ES$. In the special case in which the value of foreign currency π^* is a constant,

$$F = \frac{\pi^*}{E\pi^*/S} = \frac{1}{E1/S}.$$

Even under risk-neutrality the current forward rate is not in general precisely equal to the expected future spot rate. Indeed, under the special conditions under which they are equal from the domestic viewpoint ($F = ES$), they cannot be equal from the foreign viewpoint ($1/F \neq E1/S$). This consequence of the convexity of the inverse function and Jensen's inequality is sometimes called the Siegal paradox [Siegal (1972)]. The intuitive explanation is that people seek to maximize expected wealth in real terms rather than in terms of domestic currency, and the real value of domestic currency varies with the exchange rate (except in the special case in which the domestic price level is constant). The consensus is that the phenomenon is a mathematical inconvenience, but not a matter of economic or empirical significance.

Now we assume that people are risk-averse, i.e. $b > 0$. We consider first the case where there are no outside assets, which means that net assets in each currency, \bar{B} and \bar{B}^* , are zero, and where future total real income X is independent of the value of either the domestic or foreign currency: $E\pi^*X = E\pi^*EX$ and $E\pi X = E\pi EX$. From (2) we have

$$F = \frac{E\pi^*(1 - bEX)}{E\pi(1 - bEX)} = \frac{E\pi^*}{E\pi}.$$

This is precisely the same expression we got in the risk-neutral case. There is no risk premium. An increase in the net indebtedness of one country and net assets of the other has no effect on the current rate, not even if we assume that such debt must be denominated in the currency of the debtor country. Nor does an increase in the variability of the values of the currencies have any effect.

The reason there is no risk premium can be seen intuitively. Since the exchange rate is uncorrelated with real wealth, exchange risk is completely

diversifiable. The holder of risky foreign assets will sell foreign exchange forward; there is a corresponding holder of the risky foreign liability who will be glad to buy the foreign exchange forward. In market equilibrium, no one will be subject to risk associated with variability of the exchange rate. This completes the demonstration of proposition (1) above.

4. When there are outside assets

Now we consider the case where there are outside assets $\bar{B} > 0$ denominated in the domestic currency and outside assets $\bar{B}^* > 0$ denominated in the foreign currency. From (2):

$$F = \frac{E\pi^* - b\bar{B}E\pi\pi^* - b\bar{B}^*E\pi^{*2}}{E\pi - b\bar{B}E\pi^2 - b\bar{B}^*E\pi\pi^*} \quad (3)$$

To see the effect of an increase in \bar{B} , we differentiate. For simplicity, we assume that we are starting from a position of zero outside assets:

$$\frac{\partial F}{\partial \bar{B}} = b \frac{-E\pi\pi^*E\pi + E\pi^2E\pi^*}{[E\pi]^2} \quad (4)$$

This quantity is positive iff

$$\frac{E\pi^2}{E\pi} > \frac{E\pi\pi^*}{E\pi^*}$$

To obtain an alternative version of this condition we use the facts

$$\text{var } \pi = E\pi^2 - (E\pi)^2,$$

$$\text{cov}(\pi, \pi^*) = E\pi\pi^* - E\pi E\pi^*, \text{ and}$$

$$\text{corr}(\pi, \pi^*) = \text{cov}(\pi, \pi^*) / \sqrt{(\text{var } \pi)} \sqrt{(\text{var } \pi^*)}:$$

$$\frac{\sqrt{(\text{var } \pi)}}{E\pi} > \text{corr}(\pi, \pi^*) \frac{\sqrt{(\text{var } \pi^*)}}{E\pi^*} \quad (5)$$

The ratios on each side of the inequality represent the variability of π and

⁶The expression for F jumps from ∞ to $-\infty$ when the denominator is zero, but this is because marginal utility turns negative for the quadratic utility function. The relevant range of \bar{B} and \bar{B}^* lies below this point, so that the denominator of the expression is positive.

π^* , respectively, normalized by their means. A sufficient condition is that the variability of π is greater than the variability of π^* . Another sufficient condition is that π and π^* are not positively correlated. When either of these conditions is met, an increase in the stock of outside assets denominated in domestic currency causes the risk premium on foreign currency to fall and the current exchange rate to rise. The effect is the same as in the conventional presumption.

On the other hand, when these conditions are not met, i.e. when the value of foreign currency is highly variable and is highly correlated with the value of domestic currency, we get the surprising result that an increase in the supply of domestically denominated assets *lowers* F , the relative price of foreign-denominated assets. The explanation for this result lies in the characteristic of the quadratic utility function that it implies increasing absolute risk-aversion.⁹ An increase in \bar{B} is an increase in wealth, which causes an increase in risk-aversion and thus a shift in demand to the less risky asset.

To see the effect of an increase in foreign-denominated outside assets, we look at the derivative with respect to \bar{B}^* , again evaluated at zero for simplicity:

$$\frac{\partial F}{\partial \bar{B}^*} = b \frac{-E\pi^{*2}E\pi + E\pi\pi^*E\pi^*}{[E\pi]^2} \tag{6}$$

This quantity is negative iff

$$\frac{\text{var } \pi^*}{E\pi^*} > \text{corr}(\pi, \pi^*) \frac{\text{var } \pi}{E\pi} \tag{7}$$

An increase in foreign outside assets causes the risk premium to rise and the current exchange rate to fall, provided the variability of the domestic currency value is large relative to the variability of the foreign currency value, or else provided the currency values are not highly correlated. Again, if these conditions fail, we get the surprising result that an increase in the supply of foreign-denominated assets *raises* F , the relative price of foreign-denominated assets. Again, the explanation is that the increase in wealth causes an increase in risk-aversion and thus a shift in demand to the less risky asset.

⁹The measure of absolute risk-aversion is given by

$$-\frac{U''}{U'} = \frac{2b_i}{1 - 2b_i(\pi W_i)}$$

which increases with real wealth. See Arrow (1970).

To avoid the problem of increasing absolute risk-aversion, we can ask what happens if the asset supplies are varied with total wealth held constant. In other words, what happens if the central bank (of either country) increases \bar{B} and decreases \bar{B}^* subject to $d\bar{B} + F d\bar{B}^* = 0$? Such a policy has precisely the interpretation of intervention on the forward exchange market: if a central bank buys foreign exchange forward, it is giving the private sector foreign-denominated liabilities (the obligation to supply foreign exchange in the future) in return for domestically-denominated assets (future payment in domestic exchange) at the going market rate (F).

Of course, if the obligations of the central bank are regarded by the public as implying future tax liabilities, then they are inside assets and we are back to section 3. But if they are regarded as outside assets, then we can apply the results of this section, with $d\bar{B}^* = -(1/F)d\bar{B}$:

$$dF = \left(\frac{\partial F}{\partial \bar{B}} - \frac{1}{F} \frac{\partial F}{\partial \bar{B}^*} \right) d\bar{B}.$$

Again, we start from a position of zero outside assets, allowing us to use eq. (4) for $\partial F/\partial \bar{B}$, eq. (6) for $\partial F/\partial \bar{B}^*$, and $E\pi^*/E\pi$ for F .

$$dF = \frac{b}{E\pi} \left[E\pi^2 \frac{E\pi^*}{E\pi} + E\pi^{*2} \frac{E\pi}{E\pi^*} - 2E\pi\pi^* \right] d\bar{B}.$$

$dF/d\bar{B} > 0$ if the expression in brackets is positive. Since the expression in brackets is equal to

$$\text{var} [\pi \sqrt{(E\pi^*/E\pi)} - \pi^* \sqrt{(E\pi/E\pi^*)}],$$

it is always positive. In other words, forward market purchases of foreign exchange always raise its price, as one would expect.

Now we consider the effect of a change in the variance, at a given level of outside debt. We express eq. (3) in terms of correlations and variances rather than just expected values:

$$F = \frac{E\pi^* - b\bar{B}[\text{corr}(\pi, \pi^*)\sqrt{(\text{var } \pi)\sqrt{(\text{var } \pi^*)}} + E\pi E\pi^*] - b\bar{B}^* \text{var } \pi^* + (E\pi^*)^2}{E\pi - b\bar{B}[\text{var } \pi + (E\pi)^2] - b\bar{B}^* \text{corr}(\pi, \pi^*)\sqrt{(\text{var } \pi)\sqrt{(\text{var } \pi^*)}} + E\pi E\pi^*}.$$

First we consider the case where all outside assets are domestically denominated: $\bar{B}^* = 0$. We differentiate with respect to $\sqrt{(\text{var } \pi^*)}$, holding the expectation and the correlation constant.

$$\frac{\partial F}{\partial \sqrt{(\text{var } \pi^*)}} = \frac{-b\bar{B} \text{corr}(\pi, \pi^*) \sqrt{(\text{var } \pi)}}{E\pi - b\bar{B}[\text{var } \pi + (E\pi)^2]} < 0.$$

An increase in $\text{var } \pi$ is more complicated, but raises F if the correlation between π and π^* is low enough. If outside debt is foreign denominated, an increase in $\text{var } \pi$ raises F ; an increase in $\text{var } \pi^*$ lowers F if the correlation is low enough.

This completes the demonstration of proposition (2): the conventional presumption holds, provided foreign indebtedness is identical to the stock of outside assets issued in foreign currency. To restate the conventional presumption: an increase in foreign indebtedness (holding total wealth constant) or a decrease in the variability of the value of the domestic currency when there is net foreign indebtedness, causes a rise in the risk premium and a fall in the current forward exchange rate.

5. When real income is correlated with currency values

Now we consider the case where the value of real assets, such as human and nonhuman capital, is correlated with the value of the currencies. For simplicity, we revert to the assumption that there are no outside assets, so $\bar{B} = \bar{B}^* = 0$. From eq. (2),

$$F = \frac{E\pi^* - bE\pi^*X}{E\pi - bE\pi X},$$

where X is the sum of domestic and foreign real non-nominal wealth. Substituting $\text{cov}(\pi, X) - E\pi EX$ for $E(\pi X)$, and similarly for π^* ,

$$F = \frac{E\pi^*(1 + bEX) - b \text{cov}(\pi^*, X)}{E\pi(1 + bEX) - b \text{cov}(\pi, X)}.$$

We can substitute $\text{corr}(\pi, X) \sqrt{(\text{var } \pi)} \sqrt{(\text{var } X)}$ for $\text{cov}(\pi, X)$, and similarly for π^* :

$$F = \frac{E\pi^*(1 + bEX) - b \text{corr}(\pi^*, X) \sqrt{(\text{var } \pi^*)} \sqrt{(\text{var } X)}}{E\pi(1 + bEX) - b \text{corr}(\pi, X) \sqrt{(\text{var } \pi)} \sqrt{(\text{var } X)}}.$$

We can see again the result that when X is uncorrelated with π and π^* , $F = E\pi^*/E\pi$ as in the risk-neutral case.

In general, there is a risk premium. The current forward exchange rate is less than the expected future spot rate (i.e. there is a positive risk premium)

iff the following condition is met:

$$\text{corr}(\pi^*, X) \frac{\sqrt{(\text{var } \pi^*)}}{E\pi^*} > \text{corr}(\pi, X) \frac{\sqrt{(\text{var } \pi)}}{E\pi}.$$

Let us assume that the domestic country is the larger country, so π is more highly correlated (in absolute value) with X than is π^* . Assume also that the currency values are equally variable. Then what we might call the 'naive presumption', that there is always a positive risk premium from the viewpoint of the domestic country, is upheld if X is positively correlated with π and π^* , but is reversed if X is negatively correlated with π and π^* .

The effects of changes in the variability of the currency values can be read directly from the formula for F . When X is positively correlated with π^* , an increase in the variability of π^* causes F to fall, i.e. causes the foreign currency to depreciate. Similarly, when X is positively correlated with π , an increase in the variability of π causes F to rise, i.e. causes the domestic currency to depreciate. Thus the conventional presumption that increased variability of a currency's future value causes a fall in its current value,¹⁰ holds true when real nonfinancial wealth is positively correlated with the currency values. The reasoning is intuitively clear: holders of domestic currency, the value of which is positively correlated with other forms of wealth, must be compensated for increased variability for the same reason that the holder of a stock, the value of which is positively correlated with the market basket, must be compensated for increased variability.

But when X is *negatively* correlated with π^* , an increase in the variance of π^* causes F to *rise*, and when X is negatively correlated with π , an increase in the variance of π causes F to *fall*. The conventional presumption is reversed because a currency the value of which is negatively correlated with other forms of wealth affords a valuable opportunity for hedging against risk, much like a stock the value of which is negatively correlated with the market basket.

Which case, a positive or negative correlation between non-nominal wealth and the inverse of the price level, is more likely? Assume that the value of real assets can be represented by next period's real income. Assume, furthermore, that the correlation between total (domestic plus foreign) real income and the domestic price level is dominated by the correlation between *domestic* real income and the domestic price level.¹¹ Then the answer is given

¹⁰The proposition should perhaps be called the 'naive presumption', since the 'conventional presumption' says that the relationship between variability and the risk premium depends on whether the domestic country is a net creditor.

¹¹In a model where PPP did not always hold, an increase in the domestic price level relative to the foreign price level might lower domestic real income and yet raise foreign real income via a shift in demand. In the extreme, total real income might be unchanged, implying no

by a graph with an aggregate demand curve and aggregate supply curve describing the domestic economy. If supply shocks dominate, then real output is negatively correlated with the price level and thus positively correlated with its inverse: the conventional presumption holds. But if demand shocks dominate, then real output is positively correlated with the price level and thus negatively correlated with its inverse: the conventional presumption is reversed. Keynesian economics concentrated on demand shocks, such as fluctuations in exogenous investment or in government fiscal and monetary policy. More recently, increased attention has been paid to supply shocks, such as fluctuations in the terms of trade or the weather. The question of which shocks dominate cannot be settled a priori. Possibly the correlation is close to zero over the time range that is relevant (say 3 months), suggesting a risk premium close to zero.

6. Conclusion

The statements made in the literature regarding the risk premium on foreign exchange have here been shown to assume implicitly that a country's net indebtedness consists of outside assets denominated in its own currency. It cannot be denied that governments do issue outside assets, and that they are generally denominated in their own currency. But two questions arise:

(1) Can residents truly be indifferent to an increase in their government's indebtedness? This controversy is well known under the heading 'Are government bonds net wealth?' and will not be discussed here.

(2) If the risk premium that a government must pay on its debt issues (the amount by which the value of the proceeds is less than the expected future value of the money it will have to pay back) is significant in magnitude, then why doesn't the government denominate its debt in a basket of currencies, so as to allow foreign investors to diversify the risk away and thus obviate the need to pay a risk premium?¹² To the extent that governments (especially those of small countries) *do* denominate their debt in foreign currencies, they allow exchange risk to be diversified away. The extent to which this practice is not common suggests that exchange risk is not important.

As to correlation between currency values and real income, it appears a

correlation between X and π and thus no risk premium. But in general, one would expect the correlation between the domestic price level and domestic real income to dominate the correlation between it and foreign real income.

¹²There is a distinct issue of moral hazard: if a country or its government is heavily indebted in terms of its own currency, it may be tempted to inflate away the value of the debt. But investors are assumed to be as aware of this possibility as the government. They may demand compensation for the expected inflation, without a necessary increase in risk. If there was an increase in risk from moral hazard, it would be the same as any other increase in risk; it would be to the government's advantage to issue the debt in foreign currency so as not to have to pay a risk premium to investors.

priori as likely that the conventional presumption is reversed as that it is upheld. Increased exchange rate variability could very well mean decreased overall real risk, if the value of the currency is negatively correlated with the value of real assets.

The obvious moral is that care must be taken in reasoning from the portfolio choices facing individual risk-averse investors to general market equilibrium. But a possible second moral is that, even if we do not believe that the assumptions necessary for a zero risk premium (no true outside assets and no correlation between currency values and real output) are literally true, we may believe that they are close enough to true to make the assumption of no risk premium and perfect asset substitutability a useful simplification. If perfect markets are assumed (perfect information, rational expectations, and no transactions costs), then the additional assumption of no nondiversifiable exchange risk may be acceptable.

On the other hand, if perfect markets are not assumed, then nondiversifiable exchange risk regains its importance. Transactions costs may prevent individuals from diversifying away all risk in practice even if they could do so in theory. For example, even if a large debt of the Brazilian government to Americans is considered fully as much of a liability by Brazilian citizens as it is an asset by American citizens, so that the former would be happy to buy cruzeiros forward from the latter, it is nevertheless true that in Brazil no developed forward market in cruzeiros exists on which such a trade could take place. Given the exclusion of Brazilians from such a market, whether because of transactions costs or government controls, American investors will indeed require a risk premium in return for holding Brazilian assets if they are riskier than American assets. There remains the question why the Brazilian government would not issue debt denominated in dollars in such a situation.

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