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In Search of the Exchange Risk Premium: A Six-Currency Test Assuming Mean-Variance Optimization

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A number of authors, examining deviations of actual rates of currency depreciation from forward discounts, have claimed to find evidence of risk premiums. If these deviations are indeed partly due to risk premiums, rather than being entirely due to expectational errors, then they should be systematically related to those variables on which risk premiums are thought to depend. Finance theory tells us that if investors optimize with respect to mean and variance, the risk premium depends in a specific way on (1) the supplies of assets denominated in the various currencies, (2) the variance-covariance matrix of the rates of return, and (3) the coefficient of risk-aversion. The null hypothesis of this paper is that the coefficient of risk-aversion is zero, countries' assets are perfect substitutes, and risk premiums are nonexistent. The alternative hypothesis is that rates of return are systematically related to asset supplies by a macroeconomic portfolio-balance function. The maximum likelihood technique used dominates previous attempts to find such a relationship in that the parameters of the portfolio-balance function are constrained to depend on the variance-covariance matrix as the finance theory says they should. On the other hand, the technique also dominates previous applications of the finance theory in that the expected returns are allowed to vary over time, as the macroeconomic portfolio-balance theory says they should. The paper's finding is a statistical inability to reject the null hypothesis for a portfolio of six currencies, although this could be due to low power in the test.

In recent years there has been a great deal of interest in the portfolio-balance approach to modeling international financial behavior.[1] The investor is assumed to balance his portfolio among the assets of various countries as a function of their relative expected returns. An implication is that an increase in the supply of one country's assets requires something like an increase in its interest rate or in the expected appreciation of its currency, in order for the assets to be willingly held.

The empirical work under this approach has not been as successful as the

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theoretical work. In particular, no one has yet been able to reject statistically the hypothesis that countries' bonds are perfect substitutes, so that expected returns are equalized. The question is important because if expected returns are equalized (i.e., uncovered interest parity holds) then bond supplies do not matter for questions such as exchange rate determination, and investigation along the lines of the monetary approach would appear to be indicated instead.[2]

All of the many tests of unbiasedness in the forward exchange market are joint tests of efficiency and the absence of a risk premium (i.e., perfect substitutability). Tests of unconditional biasedness in the forward rate fail to find any such bias persisting over the length of the floating rate period.[3] Other tests, such as those for serial correlation in the prediction errors, do sometimes reject the joint null hypothesis. Some authors, such as Hansen and Hodrick (1980) and Cumby and Obstfeld (1981), interpret the results as evidence against perfect substitutability and for a risk premium.[4] To do so one must argue that the risk premium fluctuates frequently between positive and negative, in order to account for the absence of persistent unconditional bias.

If the imperfect substitutability interpretation were correct, one would expect the relative returns to be systematically related to asset supplies as required by the portfolio-balance equation. Dooley and Isard (1979) and Frankel (1982a) fail to find a significant positive relationship between expected returns and bond supplies. Of course, this failure to reject perfect substitutability between domestic and foreign bonds could be due to the low power of the test as easily as to the virtue of the null hypothesis.

Recently the application of finance theory, by which I mean the principles of expected utility maximization, to the international portfolio-balance model has given the model additional impetus. This application was pioneered by Kouri (1976b, 1977), and more recently formulated in a manner simple enough to be directly usable in macroeconomic models by Dornbusch (1982).[5] The parameters in the asset-demand functions, rather than being determined arbitrarily, are seen to depend in a simple way on the coefficient of risk-aversion and the return variance-covariance matrix.

Kouri and de Macedo (1978), de Macedo (1982), and Dornbusch (1980), have used the finance theory to estimate the optimal portfolio from statistical variances and covariances. These analyses are not intended as formal tests of the hypothesis that actual asset-demand functions are in fact equal to the optimizing portfolio-balance functions. Indeed the results would not hold up under such testing since the optimal demand is computed to be negative in some cases (such as the yen and French franc in Kouri and de Macedo's estimates), even though the actual supply is known to be positive. Nevertheless, the authors' motivation in estimating optimal portfolios is clearly to explain actual investor behavior, rather than, for example, to improve their personal investment portfolio planning.

Two more empirical finance papers require mention. Cornell and Dietrich (1978) fail to find a significant positive relationship between expected returns of currencies and their covariances ('betas') with a market portfolio, while Roll and Solnik (1977) do find such a relationship. These papers share with the estimates of optimal portfolios an important limitation. They implicitly assume that expected returns (i.e., interest rates and expected appreciation) are constant over time.[6] This assumption is inconsistent with recent exchange rate history, with the lack of persistent unconditional bias in the forward rate, and with the macroeconomic models in which one would want to use a successfully estimated portfolio-balance equation. The limitation is related to the fact that the papers make no use of data on asset supplies: changes in expected returns are the result of changes in asset supplies.

The aim of the present paper is to test whether asset demands are properly described by the portfolio-balance equation, as against the null hypothesis of perfect substitutability. The paper goes beyond previous attempts to relate expected rates of return (or exchange rates) to bond supplies, by bringing to bear the additional knowledge we have gained from the finance literature that the coefficients are related to the variance-covariance matrix. Imposing these restrictions should give us a more powerful test. On the other hand, the technique used here dominates that of previous empirical finance studies in that it uses the asset supply data. It thus allows the estimate of the expected relative return or risk premium to fluctuate over time, rather than requiring it to be constant over time.

The technique used is maximum likelihood estimation of the *arguments* of the portfolio-balance function (the expected returns) at the same time as the *parameters* of the function (the coefficient of risk-aversion and the variance-covariance matrix).[7] The likelihood for the zero value of risk-aversion turns out to be very close to the likelihood for reasonable positive values. This finding implies failure to reject the null hypothesis of perfect substitutability once again.

I. Derivation of the Portfolio-Balance Equation from Mean-Variance Optimization

In this section we derive the form that a multicurrency asset-demand equation should take if the investor maximizes a function of the mean and variance of his end-of-period real wealth.[8] The results are familiar from Kouri (1977) and Dornbusch (1980).

Let $\tilde{W}_t$ be real wealth. The investor must choose the vector of portfolio shares that he wishes to allocate to marks, pounds, yen, francs, and Canadian dollars: $x'_t \equiv [x_t^{DM}\ x_t^{£}\ x_t^{Y}\ x_t^{F}\ x_t^{C\$}]$. The residual is the share allocated to US dollars: $(1-x'_t\iota)$, where ι is a column vector of five ones. End-of-period real wealth depends on the portfolio allocation and on real returns:

$$\langle 1\rangle \qquad \begin{aligned} \tilde{W}_{t+1} &= \tilde{W}_t + \tilde{W}_t x'_t r_{t+1} + \tilde{W}_t(1-x'_t\iota)r^{\$}_{t+1} \\ &= \tilde{W}_t[x'_t(r_{t+1}-\iota r^{\$}_{t+1}) + 1 + r^{\$}_{t+1}] \end{aligned}$$

where $r'_{t+1} \equiv [r^{DM}_{t+1} \dots r^{C\$}_{t+1}]$ is a vector of the real returns realized on the five countries' assets. The real return on the jth asset is:

$$\langle 2\rangle \qquad 1 + r^j_{t+1} \equiv \frac{1+i^j_t}{(1+\pi^{\$}_{t+1})(1+\Delta s^j_{t+1})} \approx 1 + i^j_t - \pi^{\$}_{t+1} - \Delta s^j_{t+1}$$

where i^j_t is the one-period interest rate on j-bonds, $\pi^{\$}_{t+1}$ is the rate of inflation during the period for the appropriate basket of goods, expressed in dollars, and Δs^j_{t+1} is the rate of depreciation during the period of currency j against the dollar. Similarly, the real return on dollar assets is

$$\langle 3\rangle \qquad 1 + r^{\$}_{t+1} \equiv \frac{1+i^{\$}_t}{1+\pi^{\$}_{t+1}} \approx 1 + i^{\$}_t - \pi^{\$}_{t+1}$$

If we use the approximations, the five-currency vector of returns *relative* to the dollar is very simple:

$$r_{t+1} - \iota r^{\$}_{t+1} \approx i_t - \iota i^{\$}_t - \Delta s_{t+1}$$

where i_t and Δs_{t+1} are five-currency vectors. The real return on an asset relative to the dollar is simply the interest differential minus depreciation of the currency, because the inflation rate drops out. Thus,

$$\langle 4\rangle \qquad \tilde{W}_{t+1} = \tilde{W}_t[x'_t(i_t - \iota i^{\$}_t - \Delta s_{t+1}) + 1 + i^{\$}_t - \pi^{\$}_{t+1}]$$

We define α to be a vector of consumption shares allocated to goods produced by Germany, the United Kingdom, Japan, France, and Canada. The residual $[1-\alpha'\iota]$ is the consumption share allocated to US goods. Then the dollar inflation index is computed as follows:

$$\pi^{\$}_t = \alpha'(\bar{\pi}_t - \Delta s_t) + (1-\alpha'\iota)\bar{\pi}^{\$}_{US_t}$$

where elements of

$$\bar{\pi}'_t \equiv (\bar{\pi}^{DM}_{G_t}\ \bar{\pi}^{£}_{UK_t}\ \bar{\pi}^{Y}_{J_t}\ \bar{\pi}^{F}_{F_t}\ \bar{\pi}^{C\$}_{C_t}) \text{ and } \bar{\pi}^{\$}_{US_t}$$

represent the rates of inflation in the goods of the six countries, each expressed in terms of its own currency. The 'bars' represent an important assumption that is made at this point: goods prices are nonstochastic when expressed in the currency of the producing country. Only the exchange rate is uncertain.[9]

The expected value and variance of end-of-period wealth $\langle 4\rangle$, conditional on current information, are as follows:

$$\langle 5\rangle \quad E(\tilde{W}_{t+1}) = \tilde{W}_t x'_t(i_t - \iota i^{\$}_t - E\Delta s_{t+1}) + 1 + i^{\$}_t - \alpha'(\bar{\pi}_{t+1} - E\Delta s_{t+1}) - (1-\alpha'\iota)\bar{\pi}^{\$}_{US_{t+1}}$$

$$V(\tilde{W}_{t+1}) = \tilde{W}^2_t[V(-x'_t\Delta s_{t+1} + \alpha'\Delta s_{t+1})]$$

$$= \tilde{W}^2_t[(-x'_t + \alpha')\Omega(-x_t + \alpha)]$$

where we have defined the variance-covariance matrix of currency depreciation: $\Omega \equiv E(\Delta s_{t+1} - E\Delta s_{t+1})(\Delta s_{t+1} - E\Delta s_{t+1})'$.

Investors maximize a function of the mean and variance:

$$F[E(\tilde{W}_{t+1}),\ V(\tilde{W}_{t+1})]$$

We differentiate with respect to x_t, the vector of portfolio shares:

$$\frac{dF}{dx_t} = F_1\frac{dE(\tilde{W}_{t+1})}{dx_t} + F_2\frac{dV(\tilde{W}_{t+1})}{dx_t} = 0$$

$$F_1\tilde{W}_t(i_t - \iota i^{\$}_t - E\Delta s_{t+1}) + F_2\tilde{W}^2_t 2\Omega(x_t - \alpha) = 0$$

We define the coefficient of relative risk-aversion $\rho \equiv -F_2\tilde{W}_t 2/F_1$, which is assumed to be constant.[10] Then we have our result:

$$\langle 6\rangle \qquad i_t - \iota i^{\$}_t - E\Delta s_{t+1} = \rho\Omega(x_t - \alpha)$$

Equation $\langle 6\rangle$ is the portfolio-balance system that is estimated in the following section. Particularly important is the case of risk-neutrality. If $\rho = 0$ then the expected relative return on all assets is zero; perfect substitutability and uncovered interest parity hold.

Inverting equation ⟨6⟩ to solve for the portfolio shares provides further economic intuition:

$$\langle 7 \rangle \qquad x_t = \alpha + (\rho\Omega)^{-1}(i_t - i_t^{\$} - E\Delta s_{t+1})$$

The asset demands consist of two parts. The first term α represents the 'minimum-variance' portfolio. If an investor is extremely risk-averse ($\rho = \infty$), or if exchange rates are very uncertain ($|\Omega| = \infty$), the investor will hold countries' assets in the same proportions as the 'liabilities' represented by his consumption patterns. The second term represents the 'speculative portfolio'. A higher expected return on a given asset induces investors to hold more of that asset than is in the minimum-variance portfolio, to an extent limited only by the degree of risk-aversion and the uncertainty of the exchange rate. Again, in the case of risk-neutrality ($\rho = 0$), the assets become perfect substitutes and arbitrage insures that the expected relative returns are zero.

Before we proceed to estimate equation ⟨6⟩ econometrically, two points must be noted. First, the derivation of the equation was phrased in terms of the behavior of a single investor. We should have been thinking of the x variable as having a subscript representing the investor. To begin with, we assume that all investors in the market have the same expectations, the same degree of risk-aversion, and the same consumption patterns. Thus they have the same asset-demand functions ⟨7⟩, so we can simply aggregate across the entire market, and from here on think of the equations as applying to the aggregate world portfolio. We will relax this assumption in Section III.

The second point is due to Krugman. The approximations made in equations ⟨2⟩ and ⟨3⟩ contain errors that, though they may be small if the variances are small, are of the same order of magnitude as the risk premium itself. If the derivation is done rigorously, a term of $\Omega\alpha - \sigma^2/2$ must be added to equation ⟨6⟩. To the extent that the investor consumes the goods of a given country, an increase in the variance of its exchange rate will raise the expected purchasing power of *other* countries' assets due to Jensen's inequality. Appendix 1 repeats the theoretical derivation of the present section, with due attention paid to this point. The ultimate econometric result, a failure to reject the case of risk-neutrality, is unchanged.

II. Econometric Results

In the past, the stumbling block in econometric estimation of portfolio-balance equations or in testing the perfect substitutability hypothesis has been the measurement of expected returns. Here we are willing to assume that expectations are formed rationally, but require that they be conditional on current information, rather than being formed unconditionally and thus being estimable from sample means. We define ε_{t+1} to be the expectational error, which is independent of all information known at time t:

$$E\Delta s_{t+1} = \Delta s_{t+1} + \varepsilon_{t+1}, \qquad E(\varepsilon_{t+1}|I_t) = 0$$

We combine the rational expectations definition with equation ⟨6⟩:

$$\langle 8 \rangle \qquad i_t - i_t^{\$} - \Delta s_{t+1} = \rho\Omega(x_t - \alpha) + \varepsilon_{t+1}$$

Two important aspects of equation ⟨8⟩ are that (a) with the substitution of ex post depreciation for expected depreciation, all variables are observable, and that (b) the error term is independent of the right-hand side variable x_t. Thus we could estimate the system regressing the relative returns against x_t, equation by equation.[11] However, this technique would waste the information that the coefficient matrix is $\rho\Omega$ and the vector of constant terms is $-\rho\Omega\alpha$. *The key insight of this paper is that* Ω *is precisely the variance-covariance matrix of the error term, and the system should be estimated subject to this constraint.* The imposition of a constraint between coefficients and variances, as opposed to among coefficients, is unusual in econometrics, and requires Maximum Likelihood estimation.[12]

Appendix 2 derives the first-order conditions to maximize the likelihood function. Appendix 3 describes the method used to solve the (nonlinear) first-order conditions. Appendix 4 describes the data and calculation of the variables, which required some care. The time sample consisted of monthly observations from June 1973 to August 1980.

We constrain α to reflect actual consumption shares in 1977 since these data are readily available. But the question whether ρ is equal to or greater than zero is central, so it is not constrained but rather estimated simultaneously with Ω and with the time-varying vector of expected returns that will be the fitted values of the equations.

In the dimension of ρ, the likelihood function turns out to be very flat but monotonically decreasing over the relevant range ($\rho \geq 0$); its maximum occurs at $\rho = 0$. This finding constitutes a failure to reject at any desired degree of significance the hypothesis that investors are risk-neutral and that domestic and foreign assets are thus perfect substitutes.

It is important to realize the very limited nature of this claim. In the first place, the model employed here makes several simplifying assumptions. It assumes that goods prices are nonstochastic. It assumes that the relevant assets are limited to the bonds of the six countries (omitting equities, for example), and that the supplies of these bonds are properly measured. It assumes that the variance-covariance matrix is stationary. Finally, it assumes that investors optimize with respect to the mean and variance of their wealth one period at a time. (Hansen and Hodrick (1982) have tested some restrictions implied by a model of the risk premium derived from *intertemporal* optimization, and have failed to reject them.) Each of these simplifications could, in theory, invalidate the results, and it would be desirable to relax each of them in future research.

A second, and perhaps more important, respect in which the conclusions of this paper are limited is that a failure to reject the null hypothesis does not entitle us to assert the null hypothesis. In the present case, the power of the test is especially low because the likelihood function is so flat. We are also unable to reject such plausible values for ρ as 1.0 or 2.0. (The log likelihoods are 947.01 for $\rho = 0$, 946.93 for $\rho = 1.0$, and 946.85 for $\rho = 2.0$.)[13]

The reader who is *a priori* favorably disposed to the notion of a risk premium may prefer to put the technique developed in this paper to an alternative use, especially given the flatness of the likelihood function. If one believes that the risk premium exists and that asset demands depend on it according to a portfolio-balance function like equation ⟨7⟩, one can obtain the most efficient estimator of the variance-covariance matrix Ω, and thus of the parameters in the function, by imposing a plausible value of ρ and taking the maximum likelihood estimate subject to it. de Macedo (1980) and Krugman (1981) refer to the 'Samuelson presumption' that $\rho = 2$. Thus Table 1 reports the estimated parameters of the asset-demand functions $(\rho\Omega)^{-1}$ for

TABLE 1. The parameters of the asset demand function: $(\rho\Omega)^{-1}$ for the constraint $\rho=2$ with preferences assumed uniform across investors.

The demand for the assets of:	Depends on the expected return (relative to the dollar) on the assets of:				
	Germany	UK	Canada	France	Japan
Germany	0.283	−0.110	0.086	−0.168	−0.085
UK	−0.110	0.867	−0.118	−0.283	−0.146
Canada	0.086	−0.118	2.061	−0.126	−0.007
France	−0.168	−0.283	−0.126	0.842	−0.166
Japan	−0.085	−0.146	−0.007	−0.166	0.628

All coefficients have been divided by 1200 so that they can be used to multiply interest rates quoted on a percent per annum basis.

the case of $\rho=2$. By checking the signs of the off-diagonal elements we see that all pairs of currencies are substitutes, except that the Canadian dollar and the mark are complements. We can compute row sums to find the demand for the missing sixth asset, the US dollar: the mark is the weakest substitute for the dollar, and the Canadian dollar, not surprisingly, is the strongest. One must keep in mind that the speculative portfolio is given by the matrix in Table 1 multiplied by the expected relative returns, which are the fitted values of our equation, and that the total portfolio is the speculative portfolio plus the minimum variance portfolio α, which is the consumption share vector. There is not much point reporting the total portfolio demand at any particular point in time, since it is by construction precisely equal to the actual portfolio supply at that point in time.

III. The Test with Consumption Patterns Varying Across Countries

In the closed-economy Capital Asset Pricing Model, investors are often allowed to have different ρ's, with those who are less risk-averse 'insuring' those who are more risk-averse. In the international context, allowing investors to have different α's is surely the highest priority. Kouri and de Macedo (1978), Dornbusch (1982), and Krugman (1981) presume, realistically, that each country's investors have a relatively greater preference for its own goods, and thus, given equation ⟨7⟩ (and given sufficient risk-aversion; see Krugman) a relatively greater demand for its own assets. An implication is that current account imbalances that redistribute wealth from deficit countries to surplus countries will raise net world demand for the assets of the latter. Indeed this result accounts for much of the renewed popularity of the portfolio-balance model. In this section we allow residents of different countries to have different consumption patterns (α's), and thus different asset-holding preferences.

We repeat the asset-demand equation ⟨7⟩, with a subscript j denoting the country of residence of the investor whose holdings are being described (the United States, Germany, the United Kingdom, France, Japan, Canada, and the Rest of the World):

$$X_{jt}/W_{jt} \equiv x_{jt} = \alpha_j + (\rho\Omega)^{-1}(i_t - i_t^{\$} - E\Delta s_{t+1}) \qquad \langle 9 \rangle$$

We multiply each of the equations ($j=1, \ldots, 7$) by w_{jt}, defined to be the country's share (W_{jt}/W_t) of world wealth:

$$X_{jt}/W_t = \alpha_j w_{jt} + (\rho\Omega)^{-1}(i_t - i_t^{\$} - E\Delta s_{t+1})w_{jt}$$

We add up the seven equations, and use

$$\sum_{j=1}^{7} w_{jt} = 1,$$

to get our equation for the aggregate world portfolio:

$$\langle 10 \rangle \qquad x_t = \boldsymbol{\alpha} w_t + (\rho\Omega)^{-1}(i_t - i_t^{\$} - E\Delta s_{t+1})$$

where we have defined $\boldsymbol{\alpha}$ to be a matrix whose seven columns ($\alpha_j, j=1, \ldots, 7$) indicate the consumption preferences of residents of the seven countries. (Recall that there are five elements in each of the α_j, as there are in the vector $x_t \equiv X_t/W_t$.) Intuitively, the demand for a given country's asset depends positively not only on its expected relative return, but also on the wealth of those investors who have a relatively greater preference for that country's goods and thus for its assets. Notice that in the special case when all investors have the same preferences ($\alpha_1 = \alpha_2 = \ldots \alpha_7$), this equation reduces to equation ⟨7⟩; the distribution of wealth has no effect.

We invert the asset-demand equation ⟨10⟩ and add the rational expectations assumption to get the equation in regression form:

$$\langle 11 \rangle \qquad i_t - i_t^{\$} - \Delta s_{t+1} = \rho\Omega(x_t - \boldsymbol{\alpha} w_t) + \varepsilon_{t+1}$$

As far as the Maximum Likelihood Estimation problem described in Appendices 2 and 3 is concerned, this equation is the same as ⟨7⟩. The independent variable has simply changed from being a vector of asset supplies minus constants ($x_t - \alpha$) to a vector of asset supplies minus linear combinations of the distribution of wealth ($x_t - \boldsymbol{\alpha} w_t$).

As in Section II, the likelihood function is very flat and we technically cannot reject $\rho=0$. In this case, however, the likelihood function is slightly *increasing* in ρ; the log likelihoods are 987.33 for $\rho=0.0$, 987.44 for $\rho=1.0$, and 987.54 for $\rho=2.0$, and the maximum occurs somewhere above $\rho=30$. As before, we report the matrix $(\rho\Omega)^{-1}$, which tells us the responsiveness of asset demands to expected rates of

TABLE 2. The parameters of the asset demand function: $(\rho\Omega)^{-1}$ for the constraint $\rho=2$ with preferences varying across country of residence

The demand for the assets of:	Depends on the expected return (relative to the dollar) on the assets of: Germany	UK	Canada	France	Japan
Germany	0.289	−0.112	0.104	−0.185	−0.074
UK	−0.112	0.869	−0.137	−0.261	−0.141
Canada	0.104	−0.137	2.315	−0.167	0.047
France	−0.185	−0.261	−0.167	0.837	−0.187
Japan	−0.074	−0.141	0.047	−0.187	0.631

All coefficients have been divided by 1200 so that they can be used to multiply interest rates quoted on a percent per annum basis.

return, for the case $\rho=2$. The parameter estimates in Table 2 are similar to those in Table 1.

IV. Conclusion

What conclusion can be drawn from our failure to reject the null hypothesis? These results carry some weight against those who argue that the case for a risk premium has been firmly established. The existence of deviations of exchange rate changes from the forward discount, even if serially correlated, does not in itself constitute evidence for a risk premium. Only a systematic relationship between these deviations, on the one hand, and variables on which the risk premium is theoretically supposed to depend—such as asset supplies and the variance-covariance matrix—on the other hand, would constitute evidence of a risk premium. This paper has shown that no such systematic relationship exists. And while the power of the test may be low, imposing mean-variance optimization has given us greater power than previous tests.

Appendix 1

Derivation of the Krugman Version

As mentioned at the end of Section I, Krugman points out in a continuous-time model that Jensen's inequality is not merely a mathematical annoyance that can be swept away with an appeal to approximation, but is substantive to the question of how the parameters of the asset-demand functions depend on Ω.

Assume that the spot exchange rate of currency j against the dollar follows a continuous-time diffusion process in proportional terms:

$$\langle 12 \rangle \qquad \frac{dS^j}{S^j} = \delta_j dt + \sigma_j dZ$$

Kouri (1977) and Kouri and de Macedo (1978) derive the optimal portfolio allocation among k assets. When prices are nonstochastic, the vector of expected real returns relative to the dollar depends on asset supplies as follows:

$$\langle 6' \rangle \qquad E(r - \iota r^{\$}) = \rho\Omega(x - \alpha)$$

This equation would be identical to our equation ⟨6⟩ estimated in the text if one could validly approximate the real return on a bond as its interest rate minus expected currency depreciation and dollar inflation, as in equations ⟨2⟩ and ⟨3⟩.

Krugman shows (in a two-currency model) that the correct expected relative return on the assets of country j in ⟨6′⟩ is:

$$E(r^j - r^{\$}) = i^j - i^{\$} - \delta_j - \Omega^j\alpha + \sigma_j^2$$

where Ω^j is the j^{th} row of Ω that gives the variance σ_j^2 of exchange rate j and its covariances with the other dollar exchange rates. This is the same as in our equation ⟨6⟩ but for the presence of the $-\Omega^j\alpha + \sigma_j^2$ term. The intuitive explanation for the presence of this term is as follows. If all of the investor's consumption falls on the goods of a particular country ($\alpha^j = 1$), then there is no extra term for the relative return on that country's assets ($\Omega^j\alpha = \sigma_j^2$); changes in its exchange rates (the prices of foreign currencies in terms of domestic) are unambiguous indicators of changes in the real value of foreign assets, because the real value of domestic currency is certain. But an increase in the variance of exchange rate j raises the expected purchasing power of asset j over foreign goods due to Jensen's inequality. Thus, to the extent that foreign goods enter the relevant consumption basket ($\alpha^j < 1$), the variance raises the expected real return on asset j.

For empirical purposes, we must translate this result from continuous time to discrete time. By Ito's lemma, the *log* of the spot rate s^j follows a diffusion process related to that describing the *level* in equation ⟨12⟩:

$$ds^j = \mu_j dt + \sigma_j dZ \qquad \text{where} \qquad \mu_j \equiv \delta_j - \sigma_j^2/2$$

The nice thing about diffusion processes is that changes observed over a discrete unit time interval are normally distributed with known mean and variance:

$$E(s^j_{t+1} - s^j_t) = \mu_j \qquad \text{and} \qquad V(s^j_{t+1} - s^j_t) = \sigma_j^2$$

Therefore if the continuous-time rates of return are constant over the time interval,

$$\begin{aligned} i^j_t - i^\$_t - E\Delta s^j_{t+1} &= i^j_t - i^\$_t - \delta_j + \sigma_j^2/2 \\ &= E(r^j - r^\$) + \Omega^j\alpha - \sigma_j^2/2 \end{aligned}$$

Equation ⟨6′⟩ now translates into discrete time as:

$$\langle 6'' \rangle \qquad i_t - \iota i^\$_t - E\Delta s_{t+1} = \rho\Omega(x_t - \alpha) + \Omega\alpha - \sigma^2/2$$

where $\sigma^2/2$ is a vector whose j^{th} element is $\sigma_j^2/2$. This equation supports the seemingly ad hoc practice, common especially in joint tests of forward market efficiency and risk-neutrality ($\rho=0$), of defining exchange rates in logs to 'get around the Siegel paradox'. Careful treatment of Jensen's inequality gives rise only to a constant in the expected prediction error; time-varying expected prediction errors would require the existence of risk premia or the failure of rational expectations.

Unlike the Dornbusch form ⟨6⟩ estimated in the text, the Krugman form ⟨6″⟩ is not homogeneous in $\rho\Omega$. This turns out to make estimation of the Krugman form more difficult than the maximum likelihood estimation (MLE) for the simpler Dornbusch form, described in the next two appendices. Use of a 'canned' maximum likelihood package for the Krugman form produced results very similar to those for the Dornbusch form: over the range from $\rho=0$ to $\rho=30$ the maximum occurred at zero, and point estimates of the Ω^{-1} matrix were similar to those in Table 1. When the consumption preferences in equation ⟨6″⟩ are allowed to vary across countries, the equation can be aggregated in a way analogous to that in Section III for the Dornbusch form. The maximum now occurs at the more reasonable value of 13. But we are still unable statistically to reject zero, and the point estimates of Ω^{-1} are very similar to those in Table 2.

Appendix 2

MLE First-Order Conditions for Dornbusch Equation 6

In this appendix we write down the likelihood function for the Dornbusch model of the risk-premium, equation ⟨6⟩, and derive the first-order conditions for the maximization of the function, which are represented by equations ⟨16⟩ and ⟨18⟩. The derivation of ⟨16⟩ and ⟨18⟩ is described in some detail in this appendix, as is the method of solving ⟨16⟩ and ⟨18⟩ in the following appendix, because the problem is a general one. The estimation technique could be applied to other contexts such as a closed-economy model of money, bonds, and capital, in which the data might turn out to be more hospitable to the portfolio-balance hypothesis than they are in the present context.

We repeat the Dornbusch equation with the expectational error term ε_{t+1}:

$$\langle 13 \rangle \qquad z_{t+1} = \rho\Omega(x_t - \alpha) + \varepsilon_{t+1}$$

where we have defined the vector of relative ex post returns $z_{t+1} \equiv i_t - i_t^{\$} - \Delta s_{t+1}$. Assuming that the errors are normally distributed (see Note 8), the likelihood function is:

$$\langle 14\rangle \qquad L = (2\pi)^{-NT/2}|\Omega|^{-T/2} \exp -\frac{1}{2}\sum_{t=1}^{T} \varepsilon'_{t+1}\Omega^{-1}\varepsilon_{t+1}$$

where T is the number of observations and N is the number of exchange rates (five, in our case) and $\varepsilon_{t+1} = z_{t+1} - \rho\Omega(x_t - \alpha)$. To find the values of ρ and Ω that maximize L given the data, we first take the log of the likelihood:

$$\langle 15\rangle \qquad \ln L = -\frac{NT}{2}\ln 2\pi - \frac{T}{2}\ln|\Omega| - \frac{1}{2}\sum z'_{t+1}\Omega^{-1}z_{t+1}$$

$$-\frac{\rho^2}{2}\sum (x_t-\alpha)'\Omega(x_t-\alpha) + \rho\sum z'_{t+1}(x_t-\alpha)$$

From this expression it can be seen that three moment matrices are sufficient statistics:

$$\sum z_{t+1}z'_{t+1}, \qquad \sum (x_t-\alpha)(x_t-\alpha)', \qquad \text{and} \qquad \sum z'_{t+1}(x_t-\alpha).$$

We take the derivative of the log likelihood, first with respect to ρ, and set it equal to zero:

$$\langle 16\rangle \qquad \frac{\partial \ln L}{\partial \rho} = -\rho\sum (x_t-\alpha)'\Omega(x_t-\alpha) + \sum z'_{t+1}(x_t-\alpha) = 0$$

From ⟨16⟩ it can be seen that if Ω were known, the MLE of ρ would just be the GLS estimate of ρ in equation ⟨8⟩.[14,15]

More complicated is the derivative of the log likelihood with respect to Ω. Henri Theil (1971, p. 33), shows that

$$\frac{\partial \Omega^{-1}}{\partial \omega_{hk}} = -(\Omega^{-1}i_h)(i_k'\Omega^{-1})$$

where ω_{hk} is element h,k of Ω, and vectors i_h and i'_k select out column h and row k of Ω^{-1}.

It follows that

$$\frac{\partial x'\Omega^{-1}y}{\partial \omega_{hk}} = -(x'\Omega^{-1}i_h)(i'_k\Omega^{-1}y)$$

$$= -(i'_k\Omega^{-1}y)(x'\Omega^{-1}i_h)$$

This is element h,k of the matrix $\Omega^{-1'}xy'\Omega^{-1'}$. So we have

$$\frac{\partial x'\Omega^{-1}y}{\partial \Omega} = -\Omega^{-1}xy'\Omega^{-1} \text{ (Ω being symmetric).}$$

We use this, and the facts

$$\frac{\partial \ln|\Omega|}{\partial \Omega} = \Omega^{-1} \qquad \text{and} \qquad \frac{\partial (x'\Omega x)}{\partial \Omega} = xx'$$

(also from Theil (1971), equations (6.14) and (6.8), respectively), to differentiate ⟨15⟩.

$$\langle 17\rangle \qquad \frac{\partial \ln L}{\partial \Omega} = 0 = -\frac{T}{2}\Omega^{-1} + \frac{1}{2}\sum \Omega^{-1}z_{t+1}z'_{t+1}\Omega^{-1} - \frac{\rho^2}{2}\sum (x_t-\alpha)(x_t-\alpha)'$$

Pre- and post-multiplying ⟨17⟩ by Ω gives

$$\langle 18\rangle \qquad 0 = -\frac{T}{2}\Omega + \frac{1}{2}\sum z_{t+1}z'_{t+1} - \frac{\rho^2}{2}\sum \Omega(x_t-\alpha)(x_t-\alpha)'\Omega$$

which together with ⟨16⟩ characterizes the MLE.

Appendix 3

Method of Solution of MLE First-Order Condition for Ω

The coefficient of risk-aversion ρ, whose MLE first-order condition is given by ⟨16⟩, is easily estimated because it is a scalar. In this case, we simply conducted a grid search over the relevant range of ρ.

It is much less straightforward to estimate the variance-covariance matrix Ω. The problem is that the first-order condition ⟨18⟩ is quadratic in Ω, preventing a closed-form solution that might translate into a GLS framework on which one could use a packaged program. If we define the moment matrices

⟨19⟩ $$D \equiv \frac{1}{T}\sum z_{t+1}z'_{t+1} \quad \text{and} \quad C \equiv \frac{\rho^2}{T}\sum (x_t-\alpha)(x_t-\alpha)'$$

then ⟨18⟩ is

⟨20⟩ $$\Omega C\Omega+\Omega-D=0$$

The trick is to transform the quadratic system ⟨20⟩ into a system of equations *in scalars* that can be solved by the ordinary quadratic formula.[16]

As argued by C. R. Rao (1973, p. 41), since C^{-1} is symmetric positive definite, there exists a nonsingular matrix T such that $C^{-1}=T'T$, and since D is also symmetric, there exists a matrix P and a diagonal matrix of eigenvalues Λ such that

$$P'GP=\Lambda,\ P'P=I, \text{ where } G\equiv T^{-1'}DT^{-1}. \text{ Define } R\equiv T^{-1}P. \text{ Then}$$

$$R'DR=\Lambda \quad \text{and} \quad R'C^{-1}R=I, \quad \text{or} \quad C=RR'.$$

The point is to pre- and post-multiply ⟨20⟩ by R to diagonalize D:

$$R'\Omega C\Omega R+R'\Omega R-R'DR=0$$
$$R'\Omega RR'\Omega R+R'\Omega R-\Lambda=0$$

We can write this

$$Y^2+Y-\Lambda=0, \qquad \text{where } Y\equiv R'\Omega R$$

Y has multiple solutions. There will exist one that is diagonal;[17] its elements are given by a set of N quadratic equations: $Y_{ii}^2+Y_{ii}-\lambda_i=0$. In solving these equations we take the positive root: $Y_{ii}=(1+\sqrt{1+4\lambda_i})/2$. Once Y is calculated, Ω is formed as

$$\Omega=R^{-1'}YR^{-1}$$

The value of the log likelihood function is found by substituting the calculated Ω directly into ⟨15⟩.

The computer program to perform these calculations was written in Fortran by Tony Rodrigues and uses the IMSL package of mathematical subroutines. It takes as data the number of equations, the sample size, the absolute moment matrices of z_{t+1} and $x_t-\alpha$, and the sum

$$\sum z'_{t+1}(x_t-\alpha)$$

The range of ρ in the grid search is specified (in this case 0.0–30.0), but easily changed. The program calculates separately the likelihood for $\rho=0$, a special case in which

$$\Omega=\sum z_{t+1}z'_{t+1}/T$$

as can be seen in ⟨18⟩.

Appendix 4: Data

Rates of Return z_{t+1}

The rates of return used for the calculations reported were the log of the one-month forward exchange rate (domestic per dollar) minus the log of next month's spot rate. This should be equal to the logarithmic interest differential in excess of depreciation, $i_t - i_t^{\$} - \Delta s_{t+1}$, by covered interest parity. (In Frankel (1981), which is the version that uses only two currencies, those of the United States and Germany, I ran the tests trying both the forward rates and the interest rates. But one-month interest rates are not available for Japan.) The spot and forward rates for all countries are bid rates, 10 a.m., last day of month, from DRI.

US Dollar Assets

DOASST = world supply of dollar assets. Billions of dollars, end of month. Calculated as DODEBT + FEDINT − NDOLCB.

DODEBT = gross public debt of the US Treasury and other US government agencies, excluding that held by US government agencies and trust funds, i.e., held by the Federal Reserve, private domestic investors, and foreigners, at end of month (source: *Treasury Bulletin*, Table FD-1, as reported by DRI); minus two issues of 'Carter notes,' which are denominated in foreign currency: $1,595.2 million dating from December 1978 and another $1,351.5 million from March 1979 (source: Federal Reserve press release, June 1979).

FEDINT = dollars supplied by the Fed in cumulative foreign exchange intervention. Computed by $\text{FEDINT}_t = \text{FEDINT}_{t-1} + \Delta\text{FEDINT}_t$, on a benchmark of the dollar value of all US international reserve assets (gold, foreign exchange, SDRs, and IMF position) in January 1974 (source: Federal Reserve *Annual Statistical Digest 1973–77*, Table 51 [or *F. R. Bulletin*, Table 3, p. A59, e.g., June 1975]).

ΔFEDINT = intervention, equal to increases in reserves, corrected for valuation changes. Computed as change in gold holdings (there have been no valuation changes since 1973), plus change in foreign exchange holdings in dollars minus valuation change (last period's foreign exchange times the change in the dollar/mark rate; most of the holdings have been in marks during the only period in which they have been significant, i.e., since November 1978), plus change in SDRs and IMF position in dollars minus valuation change (last period's SDRs and IMF position times the change in the dollar/SDR rate; relevant since July 1974), minus new SDR allocations (nonzero only for January 1979, January 1980, and January 1981). (Source for reserve holdings through 1977: F. R. *Annual Statistical Digest 1973–77*, Table 51; 1978–81: *F. R. Bulletin*, Table 3.12. Source for dollar/SDR rate: IMF *International Financial Statistics*, line 78 bd.)

NDOLCB = holdings of dollar assets (regardless whether government securities) by foreign central banks as foreign exchange reserves. Billions of dollars, end of month. These data were available quarterly from 1970:12 to 1979:3 from the IMF. A figure for 1979:12 was taken from the IMF *Annual Report*, 1980, Tables 15 and 16. This number was derived by taking the SDR value of official holdings of dollars from table 15, and adding to it 12.784 billion SDRs, which is the SDR value of ECUs issued against US dollars; (i.e., dollars deposited by European central banks with the EMS in exchange for ECUs are treated as if still held by central banks.) The number derived was multiplied by the dollar/SDR rate to arrive at the 1979:12 value, in dollars, of official holdings of US dollars. A figure for 1980:12 was taken from the IMF *Annual Report*, 1981, Tables 20 and 21. This number was arrived at by taking the total official holdings of foreign exchange from Table 21, subtracting the residual (so the number would more closely match the number taken from Table 15 the previous year) and multiplying it by 0.628 (Table 20), which is the share of US dollars, including ECUs issued against dollars, in total reserves. This number is multiplied by the dollar/SDR rate to get the dollar value of official holdings of US dollars in 1980:12. The monthly numbers from 1971:1 to 1979:2 were arrived at by linear interpolation. The analogous thing is done for 1979:4 to 1980:11 with the numbers from the *Annual Report.*

Deutsche Mark Assets

DMASST = world supply of mark assets. Billions of marks, end of month. Calculated as DMDEBT + BBINT − NDMCB.

DMDEBT = debt of the German Federal Government, end of month. Source: *Monthly Report*, Deutsche Bundesbank, Table VIII 10.

BBINT = cumulative Bundesbank sales of mark assets for international reserves in exchange market intervention, calculated as GRES − GADJ.

GRES = net external position of the Bundesbank, valued in marks, end of month. Source: *Bundesbank's Statistical Supplements to the Monthly Report*, Series 3, Table 10, col. 21 (formerly Table 9a), under 'Netto-Auslandsposition'.

GADJ = 'balancing item to the Bundesbank's external position, an adjustment by Bundesbank every December to reflect capital gains on foreign exchange and other reserves (these numbers are also available from Table IX 6 (1), col. 12) and every January (except when zero: 1975 — 1978) to reflect new SDR allocations. These items must be taken back out of GRES so that only changes in reserves due to purchases or sales of mark assets are counted. Cumulated with a benchmark of zero in 1970:1. Source: *Bb Monthly Report*, Table IX 1, col. 7.

NDMCB = holdings of mark assets (regardless whether government securities) by foreign central banks as foreign exchange reserves. These data were available quarterly for 1970:12 to 1979:3 from the IMF. A figure for 1979:12 was taken from the IMF *Annual Report*, 1980, Table 15. A figure for 1980:12 was taken from the IMF *Annual Report*, 1981. The latter number was derived by taking the total official holdings of foreign exchange (from Table 21), subtracting the residual (so the number would more closely match the number taken from Table 15 the previous year), and multiplying it by 0.121 (from Table 20), which is the share of marks in total reserves. The numbers were then converted into marks by using the mark/dollar exchange rate (from the *IFS*, line ae) and the dollar/SDR exchange rate (from the *IFS*, at the front of the book, line 78 bd). Monthly numbers were obtained by linear interpolation.

Pound Sterling Assets

PSASST = world supply of pound assets. Billions of pounds, end of month. Calculated as PSDEBT + BEINT − NPSCB.

$PSDEBT_t$ = pound sterling debt of the British government. Computed as the cumulation of the government deficit (the negative of line 80 in the *IFS*) on a March 1973 benchmark of 37,156 million (source: *U.N. Stat. Yearbook 1977*, Public Finance Table No. 201). The government deficit was used rather than the better-known Public Sector Borrowing Requirement because the deficit 'corresponds to a negative figure of net acquisition of financial assets' while the PSBR (according to *Central Statistical Office Financial Statistics*, 2.3, col. 1) exceeds the deficit by 'net government lending to private sector and overseas' and 'other financial transactions.'

BEINT = cumulative Bank of England sales of pound assets for international reserves in exchange market intervention. Computed by $BEINT_t = BEINT_{t-1} + \Delta BEINT_t$ (UK Balance for Official Financing; source: *CSO Fin. Stats.*, HI, divided by three to get monthly numbers), on a 1973:1 benchmark of total international reserves in dollars (from IMF *IFS*, line 1 d . . d) times the pound/dollar exchange rate (from *IFS* line ag).

NPSCB = holdings of pound assets (regardless whether government securities) by foreign central banks as foreign exchange reserves. These data were available quarterly from 1970:12 to 1979:3 from the IMF. A figure for 1979:12 was taken from the IMF *Annual*

Report, 1980, Table 15. A figure for 1980:12 was taken from the IMF *Annual Report*, 1981. This latter number was derived by taking the total official holdings of foreign exchange (from Table 21), subtracting the residual (so the number would more closely match the number taken from Table 15 the previous year), and multiplying by 0.026 (from Table 20), which is the share of pounds in total reserves. The numbers were then converted into pounds by using the pound/dollar exchange rate (from the *IFS*, line ae) and the dollar/SDR exchange rate. Monthly numbers were obtained by linear interpolation.

Japanese Yen Assets

JYASST=world supply of yen assets. Billions of yen, end of month. Calculated as JYDEBT+BJINT−NJYCB.

JYDEBT=yen-denominated debt of Japanese government. Computed as the cumulation of the government deficit (the negative of line 80 in the *IFS*) on a benchmark of 7608.3 billion yen in March 1973 (from *IFS*, line 88).

BJINT=cumulative Bank of Japan sales of yen assets for international reserves in exchange market intervention. Computed by (1) the change in foreign exchange reserves corrected for valuation changes, equal to the yen/dollar exchange rate (from *IFS*, line ae) times the change in foreign exchange reserves expressed in dollars (from *IFS*, line 1.dd) under the assumption that most foreign exchange reserves are held as dollars; plus (2) the change in the IMF position and holdings of SDRs corrected for valuation changes, equal to the yen/dollar exchange rate times the difference between this period's IMF position and SDR holdings (from *IFS*, lines 1.cd and 1.bd, respectively), and last period's multiplied by the new over old dollar/SDR exchange rate, minus new issues of SDRs in January 1979, January 1980, and January 1981, of 170 million SDRs each (from the IMF's *Balance of Payments Statistics*); cumulated on a benchmark of (3) the yen value of total international reserves in March 1973, equal to the yen/dollar exchange rate times 18.125 billion dollars (from *IFS*, line 1). Changes in the stock of gold are ignored. There were some gold sales, but most of the changes in the reported gold figures are due to changes in the method of valuation (from $32 per oz. to market valuation in January 1980). One could attempt to correct for these factors, but it seems likely that errors in the correction process would outweigh the magnitude of actual gold sales. Thus, we decided to leave out gold sales entirely. (Evidence that actual gold sales were minimal is that gold holdings in ounces, reported on line 1.ad in *IFS*, are almost constant.)

NJYCB=holdings of yen assets (regardless whether government securities) by foreign central banks as foreign exchange reserves. These data were available quarterly for 1973:1 to 1979:4 from the IMF *Annual Report*, 1980, Table 15. A figure for 1980:4 was taken from the IMF *Annual Report*, 1981. This number was derived by taking the total official holdings of foreign exchange (from Table 21), subtracting the residual (so the number would more closely match the number taken from Table 15 the previous year), and multiplying by 0.032 (from Table 20), which is the share of total reserves held as yen. The numbers were converted to yen by using the yen/dollar exchange rate (from the *IFS*, line ae) and the dollar/SDR exchange rate. Monthly numbers were obtained by linear interpolation.

French Franc Assets

FFASST=world supply of franc assets, calculated as FFDEBT+BFINT−NFFCB.

FFDEBT=franc-denominated debt of French government. Computed as the cumulation of the government deficit (the negative of line 80 in *IFS*) on a benchmark of 81.71 billion francs in March 1973 (from *IFS*, line 88), minus foreign-currency-denominated debt of the government (from *IFS*, line 89b).

BFINT=cumulative Banque de France sales of franc assets for international reserves in exchange market intervention. Computed by (1) the change in foreign exchange reserves corrected for valuation changes, equal to the franc/dollar exchange rate (from *IFS*, line ae) times the change in foreign exchange reserves expressed in dollars (from *IFS*, line 1.dd) under the assumption that most foreign exchange reserves are held as dollars; plus (2) the change in the IMF position and holdings of SDRs corrected for valuation changes, equal to the franc/dollar exchange rate times the difference between this period's IMF position and SDR holdings (from *IFS*, lines 1.cd and 1.bd, respectively) and last period's multiplied by the new over old dollar/SDR exchange rate, minus new issues of SDRs in January 1979, January 1980, and January 1981 of 200 million SDRs each time (from the IMF's *Balance of Payments Statistics*); cumulated on a benchmark of (3) the franc value of total international reserves in March 1973, equal to the franc/dollar exchange rate times 11.182 billion dollars (from *IFS*, line 1). Changes in the stock of gold are ignored. There were some gold sales, but most of the changes in the reported gold figures are due to changes in the method of valuation (from $32 per oz. to market valuation in January 1980), or to the turning over of gold to the European Monetary Fund in exchange for ECUs. One could attempt to correct for these factors, but it seems likely that errors in the correction process would outweigh the magnitude of actual gold sales. (Evidence that actual gold sales were minimal is that gold holdings in ounces, reported on line 1.ad in *IFS*, are almost constant.)

NFFCB = holdings of French franc assets (regardless whether government securities) by foreign central banks as foreign exchange reserves. These data were available quarterly for 1973:1 to 1979:4 from the IMF *Annual Report*, 1980, Table 15. A figure for 1980:4 was taken from the IMF *Annual Report*, 1981. This latter number was derived by taking the total official holdings of foreign exchange (from Table 21), subtracting the residual (so the number would more closely match the number taken from Table 15 the previous year), and multiplying by 0.011 (from Table 20), which is the share of total reserves held as francs. The numbers were converted to francs by using the franc/dollar exchange rate (from *IFS*, line ae) and the dollar/SDR exchange rate. Monthly numbers were obtained by linear interpolation.

Canadian Dollar Assets

CDASST = world supply of Canadian dollar assets. Billions of Canadian dollars, end of month. Calculated as CDDEBT + BCINT. (Canadian dollars are not held as reserves by other central banks.)

CDDEBT = net debt of the Canadian federal government. From March 1973 to March 1976, CDDEBT is the gross debt of the federal government (from *IFS*, line 88) minus intragovernmental debt (from *IFS*, line 88s). After April 1976, those numbers are not kept up; CDDEBT is the cumulation of the Canadian government budget deficit (the minus of line 80 in *IFS*) on a benchmark of 36.745 billion Canadian dollars in April 1976 (the value from line 88 minus line 88s).

BCINT = cumulative Bank of Canada sales of Canadian dollar assets for international reserves in exchange market intervention. Computed by (1) the change in foreign exchange reserves corrected for valuation changes, equal to the Canadian dollar/US dollar exchange rate (from *IFS*, line ae) times the change in foreign exchange reserves expressed in US dollars (from *IFS*, line 1.dd) under the assumption that most foreign exchange reserves are held as US dollars; plus (2) the change in the IMF position and holdings of SDRs corrected for valuation changes, equal to the Canadian dollar/US dollar exchange rate times the difference between this period's IMF position and SDR holdings (from *IFS*, lines 1.cd and 1.bd, respectively) and last period's multiplied by the new over the old US dollar/SDR exchange rate, minus new issues of SDRs in January 1979, January 1980, and January 1981 of 141 million SDRs each time (from the IMF's *Balance of Payments Statistics*); cumulated on a benchmark of (3) the Canadian dollar value of total international reserves in March 1973, equal to the Canadian dollar/US dollar exchange rate times 6.152 billion US dollars (from

IFS, line 1). Changes in the stock of gold are ignored. There were some gold sales, but most of the changes in the reported gold figures are due to changes in the method of valuation (from $32 per oz. to market valuation in January 1980). One could attempt to correct for these factors, but it seems likely that errors in the correction process would outweigh the magnitude of actual gold sales. (Evidence that actual gold sales were minimal is that gold holdings in ounces, reported on line 1 ad in the *IFS*, are almost constant.)

Wealth W

The net wealth of the citizens of each of the six countries, in billions of local currency units, end of month, was calculated as the cumulation of the government deficit plus the current account surplus. The government deficit for the United States is the change in the gross public debt of the US Treasury, excluding only that held by US government agencies and trust funds (DODEBT *not* corrected for the two issues of 'Carter notes'); for Germany it is the change in federal government debt (DMDEBT); for the UK, Japan, France, and Canada it is the government deficit (the minus of line 80 in the *IFS*).

The monthly current account for each country was obtained by taking the current account for a given quarter, subtracting the balance of trade for that quarter to obtain the balance on services and transfers, dividing by three to get a monthly estimate, and adding to the balance of trade, which is available monthly. The quarterly current account for the United States comes from DRI; for Germany from the Bundesbank *Monthly Report Statistical Supplements* Series 3, Table 1, col. 11 (*Saldo der Leistungsbilanz*); for the UK, Japan, France, and Canada it is the sum of lines 77 aad, 77 abd, 77 add, 77 aed, and 77 agd in *IFS*, converted into local currency by the dollar/local currency exchange rate (from *IFS*, line rf). The monthly balance of merchandise trade for each country was taken as the difference between exports and imports (*IFS*, lines 70 and 71, respectively).

The benchmarks for the accumulation of wealth are in December 1972. They are very *ad hoc*, since accurate data on the *level* of wealth are impossible to get, and the benchmark is essentially only a constant term in the equation anyway. For the United States and Germany, the wealth benchmarks are taken from Dooley and Isard (1979, p. 24), 'estimated from end-of-1972 stocks in Federal debt, monetary bases, and net claims on foreigners'; they are 400 billion dollars for the United States and 200 billion marks for Germany. For the UK, Japan, France, and Canada, the wealth benchmarks are taken as the ratio of the dollar value of own GNP (from *IFS*) to US GNP in 1972, times US wealth in 1972, converted into own currency (by line rg in *IFS*); the numbers are 21.793 billion pounds, 31,169.6 billion yen, 344.56 billion francs, and 35.748 billion Canadian dollars, respectively.

For computing wealth shares (the lower-case w_j in the text) the absolute wealth numbers (W_j) are converted from local currency into billions of dollars (using line ae in *IFS*) and divided by aggregate world wealth, just as for computing asset supply shares (the lower-case x in the text) the absolute ASST numbers are converted into billions of dollars and divided by aggregate world wealth. Aggregate world wealth, in billions of dollars, is DOASST, plus DMASST times the exchange rate, plus PSASST times the exchange rate, etc. The share of wealth held by rest-of-world residents is one minus the other six shares; it turns out to be always negative.

Consumption Shares α

When residents of all countries are assumed to consume the same basket of goods, in Section II, the share of world consumption allocated to each country's goods was computed as its 1977 GNP divided by the total GNP of the six countries. The source was *IFS*. The shares were: Germany 0.1362, United Kingdom 0.0659, Japan 0.2061, France 0.1000, Canada 0.0433, and the United States 0.4485.

When residents of different countries were allowed to consume different baskets of goods, in Section III, the shares were computed as follows: For each of the six countries, the ratio of imports to GNP for 1973 was computed from the 1976 *IFS* (each in own currency). The share of each country's consumption falling on its *own* goods was computed as one minus this ratio. Then shares within the imports of the country were calculated by multiplying the import/GNP ratio by the ratios of bilateral exports reported by the five other countries to the country in question (for 1973, in the IMF's *Direction of Trade*, each in dollars) to the five-country total of exports to the country. For each of the six countries, total exports less the sum of exports to the other five countries was used as exports to the rest of the world. These six numbers divided by the sum of the six gives the consumption shares of residents of the rest of the world. (Recall that we assume that no goods *produced* by the rest of the world enter anyone's consumption. This is a consequence of the assumption that no R.O.W. assets are held.) The shares are given below.

Consumption of Goods Produced in:	USA	Germany	UK	France	Japan	Canada
(as a fraction of GNP) by residents of:						
USA	0.9316	0.0103	0.0067	0.0030	0.0172	0.0308
Germany	0.0528	0.7983	0.0270	0.0981	0.0179	0.0062
UK	0.0763	0.0682	0.7433	0.0494	0.0290	0.0340
France	0.0283	0.1096	0.0208	0.8336	0.0045	0.0027
Japan	0.0726	0.0091	0.0058	0.0038	0.8932	0.0157
Canada	0.1893	0.0074	0.0127	0.0040	0.0125	0.7739
R.O.W.	0.2372	0.2992	0.1334	0.1544	0.1446	0.0313

Notes

1. The portfolio-balance model was first applied to the international floating-rate economy by Branson (1976), Kouri (1976a), and Girton and Henderson (1977), followed by many others.
2. Dornbusch (1980) and Frankel (1982b) offer surveys of the two classes of models: portfolio-balance and monetary.
3. Stockman (1978) claims some evidence for nonzero risk premia from unconditional biasedness in the forward rate, but only after dividing the sample period into two-year subsamples, and only for two countries out of six.
4. Hansen and Hodrick (1982) find the time series pattern of forward rate prediction errors to be such that they are unable to reject some restrictions implied by an intertemporal model of the time-varying risk premium.
5. Of the many other contributors to this literature, some of the recent ones are Frankel (1979), Garman and Kohlhagen (1980), Krugman (1980), Hodrick (1981), Stulz (1981), and Adler and Dumas (1981).
6. Two optimal portfolio studies, von Furstenberg (1981) and de Macedo, Goldstein and Meerschwam (1982), do allow expected returns to vary gradually over time by estimating them from the time series of actual returns observed up to the period in question.
7. The technique is very similar to that used in Frankel (1981).
8. The assumption that returns are log-normally distributed is sufficient to imply that investors look only at the mean and variance. The normality assumption might be justified by an appeal to geometric Brownian motion observed at discrete intervals, and is necessary for the maximum likelihood estimation in any case.
9. This assumption is made by Krugman, but is considered only one special case by Kouri (1976b) and Dornbusch (1982). The assumption that prices are sticky at least in the short run—and in this case we are talking about one month—is common, for example, in Keynesian macroeconomics.
10. The Arrow-Pratt measure of relative risk-aversion is defined as $\rho \equiv -U''\mathcal{W}/U'$, where $U(\mathcal{W})$ is the utility function, the expectation of which is to be maximized. One can take a Taylor-series

approximation to $EU(\tilde{W})$ and differentiate it with respect to $E(\tilde{W})$ and $V(\tilde{W})$ to show that the two definitions of ρ are equivalent.

The utility function will have a constant coefficient of relative risk-aversion if it is exponential in form:

$$U(\tilde{W}) = \frac{1}{\gamma}\tilde{W}^{\gamma}, \quad \text{where } \rho = 1-\gamma.$$

(The solution to the one-period maximization problem considered here will be the correct solution to the general intertemporal maximization problem, if the utility function is further restricted to the logarithmic form, the limiting case as γ goes to zero, which implies $\rho=1$, or if events occurring during the period are independent of the expected returns that prevail in the following period. See Hodrick (1981) or Stulz (1981)).

11. Note the importance of the strong assumption that the asset-demand function ⟨7⟩ is correctly specified, so that the only source of regression error in ⟨8⟩ is the expectational error. If the asset stocks are measured with error, or if any other determinants of asset demands have been omitted, then a regression of equation ⟨8⟩ would produce estimates that are biased and inconsistent. These considerations justify special care in the calculation of the asset supply variables, described in the data appendix.
12. The idea of estimating asset demand equations by drawing the link between the matrix of coefficients of the expected returns and the variance-covariance matrix of the actual returns is not entirely new. See, for example, Parkin (1970) and Wills (1979).
13. There is good *a priori* reason to expect high variance of the error term since the regression errors are the expectational errors in predicting exchange rates, which are universally believed to be very large. (See, e.g., Mussa (1979).) At first thought, one might expect this to imply low power in the test. But it turns out that in the present problem, a high error variance implies *high* power in the test. The point can be made intuitively by considering the two-variable case in which the test statistic is a simple t-ratio, with the estimated coefficient in the numerator, and the square root of the variance σ^2 (over the total squared variation of the independent variable) in the denominator. We impose the optimizing hypothesis that the coefficient is equal to $\rho\sigma^2$. Then the t-ratio is proportional to σ, because the coefficient rises with σ^2 while the denominator rises only with σ. Thus the ability to reject zero is *high* if σ is high.

    A more rigorous argument could be made for the multiple-asset case based on the log likelihood function, equation ⟨15⟩ in Appendix 2. Inspection reveals that if the variance-covariance matrix Ω is believed large, the likelihood function should be sensitive to the choice of ρ, not flat. Only insufficient variation in $x_t-\alpha$, or insufficient correlation with z_{t+1}, could make the likelihood function insensitive to ρ. The real problem is probably the latter: a low

    $$\sum z'_{t+1}(x_t-\alpha).$$
14. I am indebted to Christophe Chamley for this point, and for much of my thinking on the MLE first-order conditions.
15. Notice from ⟨16⟩ that we will not get a positive estimate for ρ, as the theory says we should, unless z_{t+1} and $x_t-\alpha$ are positively correlated. In our sample they turn out to be negatively correlated, but z_{t+1} and $x_t-\alpha w_t$, relevant for Section III, turn out to be positively correlated.
16. I am indebted to James Demmel, Bruce Char, Beresford Parlett, and Paul Ruud for the solution to ⟨20⟩ described in Appendix 3.
17. Y is real if $I+4\Lambda>0$. Reassuringly, G is positive semi-definite because D is, so the elements of Λ, which are the eigenvalues of G, are all positive, and the condition holds.

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