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Much of the recent work on floating exchange rates goes under the name of the "monetary" or "asset" view; the exchange rate is viewed as moving to equilibrate the international demand for stocks of money, rather than the international demand for flows of goods as under the more traditional view. But within the monetary approach there are two very different models. These models have conflicting implications in particular for the relationship between the exchange rate and the interest rate.

The first model might be called the "Chicago" theory because it assumes that prices are perfectly flexible.¹ As a consequence of the flexible-price assumption, changes in the nominal interest rate reflect changes in the expected inflation rate. When the domestic interest rate rises relative to the foreign interest rate, it is because the domestic currency is expected to lose value through inflation and depreciation. Demand for the domestic currency falls relative to the foreign currency, which causes it to depreciate instantly. This is a rise in the exchange rate, defined as the price of foreign currency. Thus we get a positive relationship between the exchange rate and the nominal interest differential.

The second model might be called the "Keynesian" theory because it assumes that prices are sticky, at least in the short run. The most elegant asset-view statement of the Keynesian model is by Rudiger Dornbusch (1976b), to which this chapter owes much.² As a consequence of the sticky-price assumption, changes in the nominal interest rate reflect changes in the tightness of monetary policy. When the domestic interest rate rises relative to the foreign rate it is because there has been a contraction in the domestic money supply relative to domestic money demand without a matching fall in prices. The higher interest rate at home than abroad attracts a capital inflow, which causes the domestic currency to appreciate instantly. Thus we get a *negative* relationship between the exchange rate and the nominal interest differential.

The Chicago theory is a realistic description when variation in the inflation differential is large, as in the German hyperinflation of the 1920s to which Frenkel first applied it. The Keynesian theory is a realistic description when variation in the inflation differential is small, as in the Canadian float against the United States in the 1950s to which Mundell first applied it. The problem is to develop a model that is a realistic description when variation in the inflation differential is moderate, as it was among the major industrialized countries in the 1970s.

This chapter develops a model which is a version of the monetary approach to the exchange rate, in that it emphasizes the role of expectations and rapid adjustment in capital markets. The innovation is that it combines the Keynesian assumption of sticky prices with the Chicago assumption that there are secular rates of inflation. It then turns out that the exchange rate is *negatively* related to the nominal interest differential, but *positively* related to the expected long-run inflation differential. The exchange rate differs from, or "overshoots," its equilibrium value by an amount which is proportional to the real interest differential, that is, the nominal interest differential minus the expected inflation differential. If the nominal interest differential is high because money is tight, then the exchange rate lies below its equilibrium value. But if the nominal interest differential is high merely because of a high expected inflation differential, then the exchange rate is equal to its equilibrium value, which over time increases at the rate of the inflation differential.

The theory yields an equation of exchange rate determination in which the spot rate is expressed as a function of the relative money supply, relative income level, the nominal interest differential (with the sign hypothesized negative), and the expected long-run inflation differential (with the sign hypothesized positive). The hypothesis is readily tested, using the mark/dollar rate, against the two alternative hypotheses: the Chicago theory, which implies a positive coefficient on the nominal interest differential, and the Keynesian theory, which implies a zero coefficient on the expected long-run inflation differential.

3.1 The Real Interest Differential Theory of Exchange Rate Determination

The theory starts with two fundamental assumptions. The first, interest rate parity, is associated with efficient markets in which the bonds of different countries are perfect substitutes:

$$d = i - i^* \quad (1)$$

when i is defined as the *log* of one plus the domestic rate of interest (which is numerically very close to the actual rate of interest for normal values) and i^* is defined as the *log* of one plus the foreign rate of interest.³ If d is considered to be the forward discount, defined as the *log* of the forward rate minus the *log* of the current spot rate, then (1) is a statement of covered (or closed) interest parity. Under perfect capital mobility, that is, in the absence of capital controls and transactions costs, covered interest parity must hold exactly, since its failure would imply unexploited opportunities for certain profits.⁴ However, d will be defined as the expected rate of depreciation; then (1) represents the stronger condition of uncovered (or open) interest parity. Of course if there is no uncertainty, as in a perfect foresight economy, then the forward discount is equal to the expected rate of depreciation, and (1) follows directly. If there is uncertainty and market participants are risk averse, then the assumption that there is no risk premium, though not precluded, is a strong one.

The second fundamental assumption is that the expected rate of depreciation is a function of the gap between the current spot rate and an equilibrium rate, and of the expected long-run inflation differential between the domestic and foreign countries:

$$d = -\theta(s - \bar{s}) + \pi - \pi^* \quad (2)$$

where s is the *log* of the spot rate; π and π^* are the current rates of expected long-run inflation at home and abroad, respectively. (We can think of them as long-run rates of monetary growth that are known to the public.)⁵ The *log* of the equilibrium exchange rate \bar{s} is defined to increase at the rate $\pi - \pi^*$ in the absence of new disturbances; a more precise explanation will be given below. Equation (2) says that in the short run the exchange rate is expected to return to its equilibrium value at a rate which is proportional to the current gap, and that in the long run, when $s = \bar{s}$, it is expected to change at the long-run rate $\pi - \pi^*$. For the present, the justification for equation (2) will be simply that it is a reasonable form for expectations to take in an inflationary world. This claim will be substantiated in appendix 3A, after a price-adjustment equation has been specified by a demonstration that (2), with a specific value implied for θ , follows from the assumptions of perfect foresight (or rational expectations in the stochastic case) and stability.⁶ The rational value of θ will be seen to be closely related to the speed of adjustment in the goods market.

Combining equations (1) and (2) gives

$$s - \bar{s} = -\frac{1}{\theta}[(i - \pi) - (i^* - \pi^*)]. \quad (3)$$

We might describe the expression in brackets as the real interest differential.⁷ Alternatively, note that in the long run when $s = \bar{s}$, we must have $\bar{i} - \bar{i}^* = \pi - \pi^*$, where \bar{i} and \bar{i}^* denote the long-run, short-term interest rates.⁸ Thus the expression in brackets is equal to $[(i - i^*) - (\bar{i} - \bar{i}^*)]$, and the equation can be described intuitively as follows. When a tight domestic monetary policy causes the nominal interest differential to rise above its long-run level, an incipient capital inflow causes the value of the currency to rise proportionately above its long-run equilibrium level.

For a complete equation of exchange rate determination, it remains only to explain \bar{s} . Assume that in the long run, purchasing power parity holds:

$$\bar{s} = \bar{p} - \bar{p}^*, \quad (4)$$

where \bar{p} and \bar{p}^* are defined as the *logs* of the equilibrium price levels at home and abroad, respectively.⁹

Assume also a conventional money demand equation:

$$m = p + \phi y - \lambda i \quad (5)$$

where m , p , and y are defined as the *logs* of the domestic money supply, price level, and output. A similar equation holds abroad. Let us take the difference between the two equations:

$$m - m^* = p - p^* + \phi(y - y^*) - \lambda(i - i^*). \quad (6)$$

Using bars to denote long-run equilibrium values and remembering that in the long run when $s = \bar{s}$, $\bar{i} - \bar{i}^* = \pi - \pi^*$, we obtain

$$\begin{aligned} \bar{s} &= \bar{p} - \bar{p}^* \\ &= \bar{m} - \bar{m}^* - \phi(\bar{y} - \bar{y}^*) + \lambda(\pi - \pi^*). \end{aligned} \quad (7)$$

This equation illustrates the monetary theory of the exchange rate, according to which the exchange rate is determined by the relative supply of and demand for the two currencies. It says that in full equilibrium a given increase in the money supply inflates prices and thus raises the exchange rate proportionately, and that an increase in income or a fall in the expected rate of inflation raises the demand for money and thus lowers the exchange rate.

Substituting (7) into (3), and assuming that the current equilibrium money supplies and income levels are given by their current actual levels,¹⁰ we obtain a complete equation of spot rate determination:

$$s = m - m^* - \phi(y - y^*) - \frac{1}{\theta}(i - i^*) + \left(\frac{1}{\theta} + \lambda\right)(\pi - \pi^*). \quad (8)$$

This is the equation that is tested empirically for the deutsche mark in section 3.3.

3.2 Testable Alternative Hypotheses

Equation (8) is reproduced here with an error term:¹¹

$$s = m - m^* - \phi(y - y^*) + \alpha(i - i^*) + \beta(\pi - \pi^*) + u \quad (9)$$

where $\alpha (= -1/\theta)$ is hypothesized negative and $\beta (= 1/\theta + \lambda)$ is hypothesized positive and greater than α in absolute value. Tests of a hypothesis are always more interesting if a plausible alternative hypothesis is specified. One obvious alternative hypothesis is Dornbusch's incarnation of the Keynesian approach, in which secular inflation is not a factor. In fact the model developed in this chapter is the same as the Dornbusch model in the special case where $\pi - \pi^*$ always equals zero.¹² The testable hypothesis is $\beta = 0$.

Another—more conflicting—alternative hypothesis comes from the Chicago theory of the exchange rate attributable to Frenkel and Bilson. The variant presented by Bilson begins with a money demand equation like (5): $m - p = \phi y - \lambda i$. Subtracting the foreign version yields a relative money demand equation like (6): $(m - m^*) - (p - p^*) = \phi(y - y^*) - \lambda(i - i^*)$. Bilson then assumes that purchasing power parity always holds:

$$s = p - p^* = (m - m^*) - \phi(y - y^*) + \lambda(i - i^*) \quad (10)$$

An increase in the domestic interest rate lowers the demand for domestic currency and causes a depreciation. In terms of equation (9), α , the coefficient of the nominal interest differential, is hypothesized to be *positive* rather than negative.

The interest differential $(i - i^*)$ is viewed as representing the relative expected inflation rate $(\pi - \pi^*)$, either because international investment flows equate real rates of interest or because interest rate parity ensures that the interest differential equals expected depreciation, and purchasing power parity ensures that depreciation equals relative inflation. Thus the expected inflation differential (were it directly observable) could be put into

(10) instead of the nominal interest differential:

$$s = (m - m^*) - \phi(y - y^*) + \lambda(\pi - \pi^*) \quad (11)$$

In terms of equation (9), α is hypothesized to be zero and β to be positive, if we use a good proxy for $(\pi - \pi^*)$. Or, more generally, the hypothesis can be represented $\alpha + \beta = \lambda > 0$, $\alpha \geq 0$, $\beta \geq 0$. The relative sizes of α and β would depend on how good a proxy we have for the expected inflation differential.

Indeed Frenkel begins his analysis with the assumption of a Cagan-type money demand function, which uses the expected inflation rate rather than the interest rate:

$$m - p = \phi y - \lambda \pi \quad (12)$$

The assumption of purchasing power parity then gives equation (11) directly. Frenkel uses the expected rate of depreciation as reflected in the forward discount in place of the unobservable expected inflation differential, which, in well-functioning bond markets, would be the same as using the nominal interest differential.

The argument that the nominal interest differential is equal to the expected inflation differential is the same as that given in the derivation of equation (7), and indeed equation (11) is identical to equation (7), except that (7) is hypothesized to hold only in long-run equilibrium while (10) is hypothesized to hold always.¹³ The Frenkel-Bilson theory could be viewed as a special case of the real interest differential theory where the adjustment to equilibrium is assumed instantaneous; that is, θ is infinite, which of course is the same as α being zero.

The theory was originally tested by Frenkel on the German 1920–23 hyperinflation during which, it is argued, inflationary factors swamp everything else. In particular, variation in the expected inflation rate dwarfs variation in the real interest rate in the effect on the demand for money and thus the exchange rate. The argument is convincing; it is quite likely that the hypothesis $\alpha < 0$ would be rejected (or the hypothesis $\alpha \geq 0$ could not be rejected) if (9) were estimated on hyperinflation data. This just says that the Frenkel theory is the relevant one in the polar case when the inflation differential is very high and variable, much as the Dornbusch theory is clearly the relevant one in the polar case when the inflation differential is very low and stable.

It is the claim of the real interest differential theory that it is a realistic description in an environment of moderate inflation differentials such as has existed in the years since the beginning of generalized floating in 1973,

and that the alternative hypotheses break down in such an environment. Bilson has suggested and tested his theory for this period and has claimed that empirically it works better than any alternative theory proposed.

The various alternative hypotheses are summarized in terms of equation (9):

Keynesian Model, Dornbusch (1976c):	$\alpha < 0$	$\beta = 0$
Chicago Model, Bilson:	$\alpha > 0$	$\beta = 0$
Frenkel:	$\alpha = 0$	$\beta > 0$
Real Interest Differential Model:	$\alpha < 0$	$\beta > 0$

3.3 Econometric Findings

In this section the real interest differential theory is tested on the mark/dollar exchange rate.¹⁴ There are several good reasons for concentrating on this rate. The variation in the German-American inflation differential has been significant, as opposed to, for example, that in the Canadian-American or German-Swiss differentials. The exchange and capital markets were free from extensive government intervention in Germany and the United States, as opposed to, for example, those in the United Kingdom or Japan. In addition, the size of the German and American economies and the fact that there have been unexpectedly large upswings and downswings in the mark/dollar rate make this exchange rate the most important one to explain.

The sample used consisted of monthly observations between July 1974 and February 1978. The results were not greatly affected by the choice of monetary aggregate; only those using M_1 are reported in table 3.1. Industrial production indexes were used in place of national output, since the latter is not available on a monthly basis. Three-month money market rates were used for the nominal interest differential. The results are reported here with interest rates and expected inflation rates expressed on a "percent per annum" basis. Two kinds of proxies for the expected inflation differential were tried: past inflation differentials (averaged over the preceding year) and long-term interest differentials (under the rationale that the long-term real interest rates are equal).¹⁵ The advantage of the long-term interest differential is that it is capable of reflecting instantly the impact of new information such as the announcement of monetary growth targets. Alternative possible measures of expected inflation, such as lagged inflation rates or the forecasts of econometric modelers, have the advantage of

Table 3.1
Test of real interest differential hypothesis (Sample: July 1974–February 1978)

Technique	Constant	$m - m_t^*$	$y - y^*$	$r - r^*$	$\pi - \pi^*$	R^2	D.W.	β	No. of observ.
OLS	1.33 (.10)	.87 (.17)	-.72 (.22)	-.39 (.49)	7.16 (.68)	.80	.76		44
CORC	.80 (.19)	.31 (.25)	-.33 (.20)	-.065 (.49)	1.93 (1.12)	.91		.98	43
INST	1.39 (.08)	.96 (.14)	-.54 (.18)	-1.19 (.82)	6.86 (.57)		1.00		42
FAIR	1.39 (.12)	.97 (.21)	-.52 (.22)	-1.25 (.51)	7.35 (.83)			.46	41

Note: Standard errors are shown in parentheses.
 Definitions: Dependent Variable (log of) Mark/Dollar Rate.
 CORC = Iterated Cochrane-Orcutt.
 INST = Instrumental variables for expected inflation differential are Consumer Price Index (CPI) inflation differential (average for past year), industrial Wholesale Price Index (WPI) inflation differential (average for past year), and long-term commercial bond rate differential.
 FAIR = Instrumental variables are industrial WPI inflation differential and lagged values of the following: exchange rate, relative industrial production, short-term interest differential, and expected inflation differential. The method of including among the instruments lagged values of all endogenous and included exogenous variables, in order to insure consistency while correcting for first-order serial correlation, is attributed to Rav Fazz.

$m - m_t^*$ = log of German M_1 /U.S. M_1
 $y - y^*$ = log of German production/U.S. production
 $r - r^*$ = Short-term German-U.S. interest differential
 $(r - r^*)_{-1}$ = Short-term German-U.S. interest differential lagged
 $\pi - \pi^*$ = Expected German-U.S. inflation differential, proxied by long-term government bond differential

being more direct. The long-term government bond rate differential is the proxy used in the regressions reported here, though other proxies are used as instrumental variables. Details on the data are given in appendix 3B.

In each regression the signs of all coefficients are as hypothesized under the real interest differential model. When the single equation estimation techniques are used, the significance levels are weak, especially when iterated Cochrane-Orcutt is used to correct for high first-order autocorrelation.

But when instrumental variables are used to correct for the shortcomings of the expected inflation proxy, the results improve markedly. The coefficient on the nominal interest differential is significantly less than zero. This result is all the more striking when it is kept in mind that the null hypothesis of a zero or positive coefficient is a plausible and seriously maintained hypothesis; the Chicago (Frenkel-Bilson) hypothesis is rejected in this data sample. The coefficient on the expected long-run inflation differential is significantly greater than zero. Thus the unmodified Keynesian (Dornbusch) hypothesis is also rejected. Furthermore, as predicted by the real interest differential model the coefficient on the expected long-run inflation differential is significantly greater than the absolute value of the coefficient on the nominal interest differential.

Several other points are also notably supportive of the theory. (I concentrate on the last regression in table 3.1.) The coefficient of the relative money supply is not only significantly positive but is also insignificantly less than 1.0. The coefficient of relative production is significantly negative, and its point estimate of approximately $-.5$ suits well its interpretation as the elasticity of money demand with respect to income. The sum of the (negative) coefficient on the nominal interest differential and the coefficient on the expected inflation differential is an estimate of the semielasticity of money demand with respect to the interest rate; when expressed on a per annum basis, the estimate is 6.0, which provides another favorable cross-check.¹⁶

The point estimate of (on a "percent per quarter" basis) α is -5.4 . This implies that when a disturbance creates a deviation from purchasing power parity, $(1 - 1/5.4 =)$ 81.5 percent of the deviation is expected to remain after three months, and $(.815^4 =)$ 44.1 percent is expected to remain after one year. The estimate of θ on a per annum basis is $(-\log .441 =)$.819. Previous work on the speed of adjustment to purchasing power parity is even less definitive than estimates of money demand elasticities, but the present estimates of the expected speed of adjustment appear reasonable.¹⁷

As a final indication of the support table 3.1 provides for the real interest differential hypothesis, the R^2 s are high. Figure 3.1 shows a plot of the

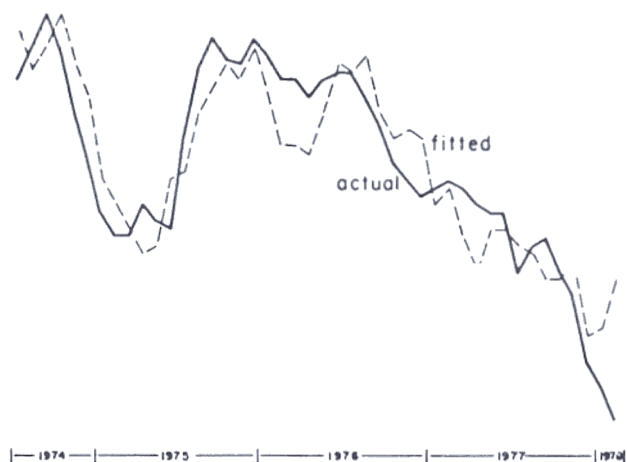


Figure 3.1
Plot of (log of) mark/dollar rate, OLS regression from table 3.1

equation's predicted values and the actual exchange rate values. The equation tracks the mark's 1974 appreciation, 1975 depreciation, and 1976–77 appreciation.¹⁸

To apply the estimated equation, let us express it as:

$$s = 1.39 + (m - m^*) - .52(y - y^*) - 1.35(i - i^*) + 7.35(\pi - \pi^*)$$

where the coefficient on the relative money supply has been set to 1.0. The expression can be decomposed into the equilibrium exchange rate

$$\bar{s} = 1.39 + (m - m^*) - .52(y - y^*) + 6.00(\pi - \pi^*)$$

and the size of the overshooting

$$s - \bar{s} = -1.35[(i - \pi) - (i^* - \pi^*)].$$

As an illustration, let us conduct the hypothetical experiment of an unexpected 1 percent expansion in the U.S. relative money supply. If the monetary expansion is considered a once-and-for-all change, then the equilibrium mark/dollar rate decreases by 1.0 percent. But in the short run the expansion also has liquidity effects; the interest semielasticity of 6.00

implies a fall in the nominal interest rate of $(1 \text{ percent}/6.00 =)$ 17 basis points.¹⁹ This fall in the real interest differential induces an incipient capital outflow, which in turn causes the currency to depreciate further, until it overshoots its new equilibrium by $(1.35 \times .17 \text{ percent} =)$.23 percent. The total initial depreciation is 1.23 percent.

This calculation assumes no change in the expected inflation rate. If the monetary expansion signals a new higher target for monetary growth, then the effect could be much greater.²⁰ Suppose the annualized 12 percent increase raises the expected inflation rate by, say, 1 percent per annum. Then there will be an additional depreciation of 6.00 percent on account of the lower demand for money in long-run equilibrium plus 1.35 percent more overshooting on account of the further reduced real interest differential. Thus the *total* initial depreciation would be 8.58 percent, of which 7.00 percent represents long-run equilibrium and 1.58 percent represents short-run overshooting.

After the initial effects, the system moves toward the new equilibrium as described in appendix 3A, provided capital is perfectly mobile and future money supplies do not deviate from their expected values. American goods are cheaper than German goods; higher demand will gradually drive up American prices faster than the rate of monetary growth, which in turn will drive up U.S. nominal interest rates, reduce the overshooting, and cause the spot rate to rise back toward its new equilibrium. After a year, approximately 44 percent of the initial real interest differential and purchasing power parity deviation will have been closed. In the meantime, there should be an expansionary effect on demand for U.S. output; lower real U.S. prices will stimulate net exports and lower real U.S. interest rates will stimulate investment. However, any effects on output have not been modeled in this chapter.²¹

3.4 Econometric Extensions

It is possible that adjustment in capital markets to changes in the interest differential is not instantaneous and that lagged interest differentials should be included in the regressions. Formally, we could argue that due to transactions costs, the forward discount adjusts fully to the interest differential with a one-month lag:

$$d = h(i - i^*) + (1 - h)(i - i^*)_{-1} \quad (13)$$

When (13) is used in place of (1), the spot rate equation (8) is replaced by

$$s = (m - m^*) - \phi(y - y^*) - (h/\theta)(i - i^*) - (1 - h)(1/\theta)(i - i^*)_{-1} + ((1/\theta) + \lambda)(\pi - \pi^*) \quad (14)$$

[The results of regressions with a lagged interest differential were reported in a table in the original version, omitted here to save space.] The coefficient on the lagged interest differential is insignificantly less than zero. This evidence supports the idea that capital is perfectly mobile.

There are several reasons why one might wish to constrain the coefficient on the relative money supply to be 1.0 in these regressions, in effect moving the relative money supply variable to the left-hand side of the equation. First, our a priori faith in a unit coefficient is high. It is hard to believe that the system could fail in the long run to be homogeneous of degree zero in the exchange rate and relative money supply. Second, errors in the money demand equation are known to have been large over the last few years. Such errors, since they are correlated with the money stock variable, would bias the coefficient downward; indeed, the coefficient estimate in one regression in table 3.1 appears significantly less than 1.0. By constraining the money supply coefficient to be 1.0, we make sure that any possible errors in the money demand equations will go into the dependent variable, that is, will be uncorrelated with any of the independent variables. Thus they cannot bias the coefficients on the interest and expected inflation differentials, which are our primary objects of concern. A third reason for constraining the coefficient is to remove the simultaneity problem which otherwise occurs if central banks vary their money supplies in response to the exchange rate. The argument even extends to direct exchange market intervention, which has been prevalent under managed floating. The right-

Table 3.2
Constrained coefficient on relative money supplies (sample: July 1974–February 1978)

Technique	Constant	$y - y^*$	$i - i^*$	$\pi - \pi^*$	R^2	D.W.	β
OLS	1.40 (.02)	-.69 (.21)	-.44 (.48)	7.54 (.42)	.92	.79	
CORC	1.16 (.13)	-.41 (.22)	-.39 (.53)	2.53 (1.21)	.96		.98
INST	1.41 (.01)	-.52 (.18)	-1.21 (.41)	6.93 (.37)		1.01	
FAIR	1.43 (.03)	-.31 (.26)	-1.40 (.68)	8.50 (1.06)			.69

Note: Standard errors are shown in parentheses. Dependent Variable: (log of) Mark/Dollar Rate \times German M_1 /U.S. M_1 . See table 3.1 for definitions.

hand-side variables determine relative money demand; changes in money demand can be reflected in either money supplies (the monetary approach to the balance of payments) or the exchange rate (the monetary approach to the exchange rate), depending on government intervention policy.

Table 3.2 reports the constrained regressions. The results are very similar to those in table 3.1. The R^2 s indicate that over 90 percent of the variation in the dependent variable is explained; the remaining 10 percent could be attributed to errors in the two countries' money demand equations.

3.5 Summary

The model developed in this chapter is a version of the asset view of the exchange rate, in that it emphasizes the role of expectations and rapid adjustment in capital markets. It shares with the Frenkel-Bilson (Chicago) model an attention to long-run monetary equilibrium. A monetary expansion causes a long-run depreciation because it is an increase in the supply of the currency, and an increase in expected inflation causes a long-run depreciation because it decreases the demand for the currency.

On the other hand, the model shares with the Dornbusch (Keynesian) model the assumption that sticky prices in goods markets create a difference between the short run and the long run. When the nominal interest rate is low relative to the expected inflation rate, the domestic economy is highly liquid. An incipient capital outflow will cause the currency to depreciate, until there is sufficient expectation of future appreciation to offset the low interest rate. The exchange rate overshoots its equilibrium value by an amount proportional to the real interest differential.

The real interest differential model includes both the Frenkel-Bilson and Dornbusch models as polar special cases. When the spot rate equation (8) is econometrically estimated for the mark/dollar rate from July 1974 to February 1978, the evidence clearly supports the model against the two alternatives.

Appendix 3A: The Price Equation and the Path to Equilibrium

In this appendix we examine the consequences of an additional assumption, a price equation. Unless there is some stickiness in p , it cannot differ from \bar{p} , and thus the domestic real interest rate cannot differ from the foreign real interest rate, or the exchange rate from the relative price level. This sticki-

ness can be embodied in the assumption that prices are fixed at a moment in time but move gradually toward equilibrium. In an environment of secular monetary growth, it is necessary that when prices reach their equilibrium, they are increasing at the secular rate. The simplest possible price equation meeting these requirements is $Dp = \delta(s - p + p^*) + \pi$, using D for the time derivative operator,

$$Dp = \delta(s - p + p^*) + \pi. \quad (A1)$$

This equation can be rationalized by expressing the rate of change of prices as the sum of a mark-up term π , representing the pass-through of domestic cost inflation and an excess demand adjustment term, where excess demand is assumed a function of the purchasing power parity gap $(s - p + p^*)$.²² Assuming that the analogous equation holds abroad, the relative price level changes according to

$$D(p - p^*) = \delta(s - p + p^*) + \pi - \pi^* \quad (A2)$$

where δ has been redefined to be the sum of the domestic and foreign adjustment parameters.

The purchasing power parity gap (also called the real exchange rate) can be shown to be proportional to the real interest differential. Substituting (6) into (7) implies

$$\bar{s} = p - p^* - \lambda[(i - \pi) - (i^* - \pi^*)]$$

which with (3) implies

$$s - p + p^* = -\left(\frac{1}{\theta} + \lambda\right)[(i - \pi) - (i^* - \pi^*)]. \quad (A3)$$

Now we use (A2) to solve out $(\pi - \pi^*)$, and collect terms to arrive at the promised result:

$$s - p + p^* = -\frac{1 + \lambda\theta}{\theta - (1 + \lambda\theta)\delta}[(i - Dp) - (i^* - Dp^*)]. \quad (A4)$$

Let us now proceed to derive the path from the initial point after a disturbance (short-run equilibrium) to long-run equilibrium. We already know from equations (3) and (A3) that the gap between s and its equilibrium and the purchasing power parity gap are each proportional to $[(i - \pi) - (i^* - \pi^*)]$, so they must be proportional to each other:

$$(s - p + p^*) = (1 + \lambda\theta)(s - \bar{s}). \quad (A5)$$

Using $\bar{s} - \bar{p} + \bar{p}^* = 0$ and (A5),

$$\begin{aligned} s - p + p^* &= (s - \bar{s}) - (p - \bar{p}) + (p^* - \bar{p}^*) \\ &= -\frac{1 + \lambda\theta}{\lambda\theta}[(p - p^*) - (\bar{p} - \bar{p}^*)]. \end{aligned} \quad (A6)$$

Substituting (A6) into (A2),

$$D(p - p^*) = -\delta(1 + \lambda\theta)/\lambda\theta[(p - p^*) - (\bar{p} - \bar{p}^*)] + \pi - \pi^*. \quad (A7)$$

This differential equation has the solution

$$(p - p^*)_t = (\bar{p} - \bar{p}^*)_t + \exp[-\delta(1 + \lambda\theta)/\lambda\theta t][(p - p^*)_0 - (\bar{p} - \bar{p}^*)_0]. \quad (A8)$$

The relative price level moves toward its equilibrium at a speed $\delta(1 + \lambda\theta)/\lambda\theta$ is proportional to the gap. The equilibrium relative price level, it must be remembered, is itself increasing at the rate $\pi - \pi^*$.

An analogous equation holds for s . Equations (A5) and (A6) tell us

$$s - \bar{s} = -\frac{1}{\lambda\theta}[(p - p^*) - (\bar{p} - \bar{p}^*)]. \quad (A9)$$

Taking the time derivative,

$$\begin{aligned} Ds &= -\frac{1}{\lambda\theta}D[(p - p^*) - (\bar{p} - \bar{p}^*)] + D\bar{s} \\ &= -\frac{\delta(1 + \lambda\theta)}{\lambda\theta}(s - \bar{s}) + \pi - \pi^*. \end{aligned} \quad (A10)$$

This differential equation has the solution

$$s_t = \bar{s}_t + \exp[-(\delta(1 + \lambda\theta)/\lambda\theta)t](s - \bar{s})_0. \quad (A11)$$

Comparing (A10), the expression for the rate of change of the spot rate if there are no further disturbances, with (2), the expression for the expected rate of change of the exchange rate, we see that the two are of the same form. Perfect foresight (or rational expectations in the stochastic case) holds if $\theta = \delta(1 + \lambda\theta)/\lambda\theta$, which has the solution

$$\begin{aligned} \bar{\theta}_1 &= \frac{\delta\lambda + ((\delta\lambda)^2 + 4\delta\lambda)^{1/2}}{2\lambda} \\ \bar{\theta}_2 &= \frac{\delta\lambda + ((\delta\lambda)^2 + 4\delta\lambda)^{1/2}}{2\lambda}. \end{aligned} \quad (A12)$$

Here we throw out the negative root because θ was assumed positive when

(2) was specified. We can see that $\bar{\theta}_1$ increases with δ , the speed of adjustment in goods markets. In turn, we know from equation (3) that the sensitivity of the exchange rate to monetary changes decreases with θ . The implication is that the slower is adjustment in the goods market, the more volatile must the exchange rate be in order to compensate.

It is easy to show that we could have derived (2) from the rest of the model and the assumptions of perfect foresight and stability, instead of assuming the form of expectations directly. Substituting the relative money demand equation (6) into the interest parity condition (1),

$$\begin{aligned} d &= (1/\lambda)[(p - p^*) - (m - m^*) + \phi(y - y^*)] \\ &= (1/\lambda)[(p - p^*) - (\bar{p} - \bar{p}^*)] + \pi - \pi^*. \end{aligned} \quad (\text{A13})$$

The perfect foresight assumption is $d = Ds$. Equation (A13) and the price equation (A2) can be represented in matrix form:

$$\begin{bmatrix} Ds \\ D(p - p^*) \end{bmatrix} = \begin{bmatrix} 0 & 1/\lambda \\ \delta & -\delta \end{bmatrix} \begin{bmatrix} s \\ (p - p^*) \end{bmatrix} + \begin{bmatrix} -(1/\lambda)(\bar{p} - \bar{p}^*) + \pi - \pi^* \\ \pi - \pi^* \end{bmatrix}.$$

Let $-\theta_1$ and $-\theta_2$ be the characteristic roots:

$$\begin{vmatrix} \theta & 1/\lambda \\ \delta & -\delta + \theta \end{vmatrix} = -\delta\theta + \theta^2 - \delta/\lambda = 0.$$

The solution is given by (A12). The path of s is given by

$$(s - \bar{s})_1 = a_1 \exp(-\theta_1 t) + a_2 \exp(-\theta_2 t)$$

The system is stable if and only if $a_2 = 0$, which, with the initial condition $a_1 = (s - \bar{s})_0$, implies equation (2), and the positive root from (A12).

Appendix 3B

The data are as follows:

Spot rate: Monthly averages of dollars per mark, *Federal Reserve Bulletin (FRB)*.

Money supply: Germany: Position at end of month, seasonally adjusted, in billions of marks, Deutsche Bundesbank (DB). United States: Averages of daily figures, seasonally adjusted, in billions of dollars, from the *Economic Report of the President (ERP)*, FRB, and *Economic Indicators (EC)*.

Industrial production: Germany: Seasonally adjusted, DB. United States: Seasonally adjusted, ERP and *Statistical Releases*.

Short-run interest rates: Representative bond equivalent yields on major 3–4-month money market instruments, excluding Treasury Bills, *World Financial Markets (WFM)*.

First expected inflation rate: Long-term government bond yields: at or near end of month, WFM. Long-term commercial bond yields: at or near end of month, WFM.

Wholesale price index (average logarithmic rate of change over preceding year): Germany: Industrial products, seasonally adjusted, DB. United States: Industrial, seasonally adjusted, ERP, FRB, *International Financial Statistics (IFS)*, and *Business Week*.

Consumer price index (average logarithmic rate of change over preceding year): Germany: Cost of living, seasonally adjusted, DB. United States: Urban dwellers and clerical workers, ERP, IFS, and EC.

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