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Expectations and Commodity Price Dynamics: The Overshooting Model

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Monetary policy has important effects on agricultural commodity prices because, though they are flexible, other goods prices are sticky. This paper formalizes the argument by applying the Dornbusch overshooting model. A decline in the nominal money supply is a decline in the real money supply in the short run. It raises the real interest rate, which depresses real commodity prices. They overshoot their new equilibrium in order to generate an expectation of future appreciation sufficient to offset the higher interest rate. These real effects (which vanish in the long run) also result from a decline in the money growth rate.

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When considering the determination of agricultural commodity prices, it is increasingly difficult to ignore the role of macroeconomic and financial factors. Schuh (1974) first pointed out the importance of these factors. But in many of the papers that followed, the exchange rate was the sole mechanism of transmission from monetary policy to agricultural commodity prices. (Besides Schuh 1974, 1976, see Chambers and Just 1981, 1982.) The importance of the exchange rate, especially clear in recent years, should not obscure the point that monetary policy has effects on the real prices of agricultural commodities even in a closed economy. An increase in the expected economy-wide inflation rate due, for example, to an increase in the money growth rate causes investors to shift out of money and into commodities. As a consequence of the increased demand for commodities, expected future inflation has a positive effect on commodity prices in the present. On the other hand, an increase in the nominal interest rate in excess of the expected inflation rate (that is, an increase in the real interest rate) due, for example, to a decrease in the level of the money supply or to a fiscal expansion causes investors to shift out of commodities and into bonds. It thus has a negative effect on commodity prices.

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This note lays out a simple model which captures these effects. The model is a very direct application of the overshooting model of exchange rates developed by Dornbusch. Dornbusch emphasized the distinction between the prices of foreign currencies, which are free to adjust instantly in response to changes in supply or demand, and the prices of most goods and services, which are not. In this paper we simply substitute the prices of basic commodities for the prices of foreign currencies. We also allow for changes in the trend rate of money growth, in addition to the changes in the money supply level that were considered by Dornbusch.

Bosworth and Lawrence (pp. 77–87), Frankel (pp. 560–63), and—the classic reference—Okun discuss the implications of, and possible reasons for, the tendency of commodities to have flexible prices much like assets while other goods and services have sticky prices. Bordo has shown empirically that the prices of raw goods indeed respond more quickly to changes in the money supply than do prices of manufactured goods.

Consider first an unanticipated one percent drop in the money supply that is expected to be permanent. In the long run we would expect all prices, manufactured goods as well as commodities, to fall by one percent in the absence of new disturbances. But in the short run manufactured prices are fixed. Thus, the reduction in the nominal money supply is a reduction in the real money supply. To equili-

brate money demand, interest rates of course rise. But we have an arbitrage condition that must hold in the commodity markets: since commodities are storable, the rate of return on Treasury bills can be no greater than the expected rate of increase of commodity prices plus storage costs. This means that the spot price of commodities must fall today and must fall by more than the one percent that it is expected to fall in the long run. In other words, commodities prices must overshoot their long-run value. Only then can there be a rational market anticipation of future capital gain that is sufficient to offset the higher interest rate.

Consider now an alternative experiment: an unanticipated increase in the expected long-run rate of money growth with no change in the current actual money supply. Of course the rate of increase of all prices, manufactured goods as well as commodities, will in the long run be equal to the new rate of money growth in the absence of new disturbances. (We are taking secular growth in real income and in velocity as exogenous and, for simplicity, equal to zero.) In the long run the inflation rate will be built into a high nominal interest rate.[1] But in the short run the nominal interest rate does not rise fully to reflect the higher inflation rate. The real interest rate falls. Now recall the arbitrage condition that precludes a difference between the interest rate and the expected rate of increase of commodity prices less storage costs. At the moment of the increase in the expected rate of money growth, commodity prices must jump up above their long-run equilibrium path. Only then can there be a rational market anticipation of future depreciation (relative to the long-run inflation rate in the economy) that is sufficient to offset the lower (real) interest rate. Thus we have overshooting of equilibrium in this case as well.

We now turn to the model of determination of commodity prices that formalizes this notion of overshooting in response to changes in the expected level or growth rate of the money supply.

We define two prices, the price of basic commodities, p_c in log form, and the price of manufactures, p_m in log form. For simplicity we are aggregating all commodities together. Commodities are homogenous and storable and thus subject to the condition that their expected rate of change $\dot{p}_c^e$ minus storage costs sc, is equal to the short-term nominal interest rate i:

$$(1) \qquad i = \dot{p}_c^e - sc.$$

(We assume that the risk premium is either equal to zero or is subsumed in the storage costs, which are assumed constant.) It will turn out that the level of p_c is determined by equation (1) together with the rest of the model and the assumption that expectations are rational.

Unlike the commodities, the level of manufacture prices is fixed by its own past history. It can adjust in response to excess demand only gradually over time, in accordance with an expectations-augmented Phillips curve:

$$(2) \qquad \dot{p}_m = \pi(d - \bar{y}_m) + \mu$$

where d is the log of demand for manufactures, $\bar{y}_m$ is the log of potential output in that sector, and μ is a term representing the expected secular rate of inflation. Here we can think of μ as the expected rate of money growth.[2] Excess demand is in turn defined as an increasing function of the price of commodities relative to manufactures and a decreasing function of the real interest rate:[3]

$$(3) \quad d - \bar{y}_m = \delta(p_c - p_m) - \sigma(i - \mu - \bar{r}).$$

We can think of $\bar{r}$ as any constant term. But our definition of long-run equilibrium will be zero excess demand ($\bar{d} = \bar{y}_m$). Thus, in long-run equilibrium the relative price of the two commodities ($p_c - p_m$) settles down to a given value ($\bar{p}_c - \bar{p}_m$), for convenience normalized at zero in log form, and the real interest rate ($i - \mu$) settles down to the given constant value $\bar{r}$.

We substitute (3) in (2):

$$(4) \quad \dot{p}_m = \pi[\delta(p_c - p_m) - \sigma(i - \mu - \bar{r})] + \mu.$$

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[1] Furthermore, the higher interest rate implies a fall in real money demand in the long run. With no jump in the current level of the money supply (as opposed to its growth rate), the long-run equilibrium path of the price level must shift up discretely (in addition to becoming steeper) in order to reduce the equilibrium real money supply. In the exchange rate literature, e.g., Frenkel, this is sometimes called the "magnification effect." See equation (17) below.

[2] The model is qualitatively unchanged if we adopt other interpretations of μ such as the rate of change of $\dot{p}_m$ or $\dot{p}$ defined below. See Obstfeld and Rogoff or Engel and Frankel (1984).

[3] The description of $i - \mu$ as the real interest rate is loose because i is the short-term interest rate while μ is the expected long-term inflation rate. However, the model is again qualitatively unchanged if we substitute the expected short-term inflation rate $\dot{p}^e$. See, for example, Obstfeld and Rogoff.

The last sector of our model is the money market. We assume a simple money demand equation:

$$(5) \qquad m - p = \phi y - \lambda i$$

where m is the log of the nominal money supply, p is the log of the overall price level, y is the log of total output, ϕ is the elasticity of money demand with respect to output, and λ is the semielasticity of money demand with respect to the interest rate. The overall price level is an average of manufacture prices, with weight α, and commodity prices, with weight $(1 - \alpha)$:

$$(6) \qquad p = \alpha p_m + (1 - \alpha)p_c.$$

Substituting in (5),

$$(7) \quad m - \alpha p_m - (1 - \alpha)p_c = \phi y - \lambda i.$$

We now consider the long-run equilibrium version of the money demand equation:

$$(8) \quad \bar{m} - \alpha \bar{p}_m - (1 - \alpha)\bar{p}_c = \phi \bar{y} - \lambda \bar{i}, \\ = \phi \bar{y} - \lambda(\bar{r} + \mu)$$

where we have used our result that the long-run real interest rate $\bar{i} - \mu$ is $\bar{r}$.

We take the difference of the two equations (7) and (8),

$$(9) \quad \alpha(p_m - \bar{p}_m) + (1 - \alpha)(p_c - \bar{p}_c) \\ = \lambda(i - \mu - \bar{r}),$$

where we have assumed that there are no expected changes in the money supply ($m = \bar{m}$) other than the expected rate of constant growth, and we have for simplicity here taken output to be fixed at the level of potential output:[4] $y = \bar{y}$.

Now we bring the different components of our model together. We combine equations (1) and (9):

$$(10) \quad \dot{p}_c^e = \frac{\alpha}{\lambda}(p_m - \bar{p}_m) \\ + \frac{(1 - \alpha)}{\lambda}(p_c - \bar{p}_c) + \mu + \bar{r} + sc.$$

We also combine equations (4) and (9) (and use the normalization $\bar{p}_c - \bar{p}_m = 0$):

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[4] The assumption that output is fixed means that the excess demand referred to in equations (1) and (2) must be coming out of inventories. It would be preferable to have manufactured output endogenously determined by demand: $y_m = d$ (and $y = y_m + \bar{y}_c$). Once again, the model is not qualitatively altered by such an extension. See the appendix to Dornbusch.

$$(11) \quad \dot{p}_m = \pi\{\delta[(p_c - \bar{p}_c) - (p_m - \bar{p}_m)] \\ - \sigma/\lambda[\alpha(p_m - \bar{p}_m) \\ + (1 - \alpha)(p_c - \bar{p}_c)]\} + \mu \\ = -\pi[\delta + \sigma\alpha/\lambda](p_m - \bar{p}_m) \\ + \pi[\delta - \sigma(1 - \alpha)/\lambda](p_c - \bar{p}_c) \\ + \mu.$$

We close the model by assuming that expectations are formed rationally: $\dot{p}_c = \dot{p}_c^e$. Equations (10) and (11) can be represented in matrix form:

$$(12) \quad \begin{bmatrix} \dot{p}_m \\ \dot{p}_c \end{bmatrix}$$

$$= \begin{bmatrix} -\pi(\delta + \sigma\alpha/\lambda) & \pi(\delta - \sigma(1 - \alpha)/\lambda) \\ \alpha/\lambda & (1 - \alpha)/\lambda \end{bmatrix}$$

$$\cdot \begin{bmatrix} (p_m - \bar{p}_m) \\ (p_c - \bar{p}_c) \end{bmatrix} + \begin{bmatrix} \mu \\ \mu + \bar{r} + sc \end{bmatrix}.$$

The characteristic roots for (12) are the solutions $-\theta_1$ and $-\theta_2$ to

$$(13) \quad [-\pi(\delta + \sigma\alpha/\lambda) + \theta][(1 - \alpha)/\lambda + \theta] \\ - (\alpha/\lambda)\pi[\delta - \sigma(1 - \alpha)/\lambda] = 0 \\ -\theta = [-(1 - \alpha)/2\lambda + \pi(\delta + \sigma\alpha/\lambda)/2] \\ \pm \sqrt{[-(1 - \alpha)/2\lambda + \pi(\delta + \sigma\alpha/\lambda)/2]^2 + \delta\pi/\lambda}.$$

The solutions for the expected future paths of the two prices in level form, as τ goes from 0 to ∞, are

$$(14) \quad p_m(\tau) - \bar{p}_m(\tau) \\ = \exp(-\theta\tau)[p_m(0) - \bar{p}_m(0)] \text{ and} \\ p_c(\tau) - \bar{p}_c(\tau) \\ = \exp(-\theta\tau)[p_c(0) - \bar{p}_c(0)]$$

where $-\theta$ is the negative root from (13). (We have thrown out the positive root to insure stability.)

In rate-of-change form the equations are[5]

$$(15) \quad \dot{p}_m = -\theta(p_m - \bar{p}_m + \mu \\ \dot{p}_c = -\theta(p_c - \bar{p}_c) + \mu + \bar{r} + sc.$$

Notice that in the special case in which manufactured prices are perfectly flexible (π, their

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[5] Notice that the secular inflation term in $\dot{p}_c$ exceeds that in $\dot{p}_m$ by $\bar{r} + sc$. This is a general problem with the commodity arbitrage condition (1). There are two possibilities. First, for an agricultural commodity, $\bar{p}_c$ may gradually increase relative to $\bar{p}_m$ (monetary considerations aside) during most of the year, as long as some of the previous harvest peak is being stored, and fall discontinuously when the new harvest comes in. (In anticipation, the stocks held would dwindle to zero before the harvest.) Thus there is no long-run trend in $\bar{p}_c - \bar{p}_m$. Alternatively, for a nonperishable, nonrenewable commodity such as gold or oil, there may indeed be a long-run trend in $\bar{p}_c - \bar{p}_m$, à la Hotelling. I am grateful to Peter Berck and Rudiger Dornbusch for both of these explanations.

responsiveness to excess demand, is infinite), θ is infinite, and the entire system adjusts to its long-run equilibrium instantaneously.

Most of the preceding was simply to establish that the rationally expected rate of change of commodities prices takes the simple regressive form of (15). Combining with the arbitrage condition (1),

$$(16) \qquad p_c = \bar{p}_c - \frac{1}{\theta}(i - \mu - \bar{r}).$$

Notice that if a change in macroeconomic policy has raised (lowered) the real interest rate $i - \mu$ above (below) its long-run equilibrium level $\bar{r}$, then commodity prices p_c have fallen below (risen above) their long-run equilibrium path $\bar{p}_c$. It is necessary that commodities be currently "undervalued" (overvalued) so that there will be an expected future rate of increase (decline) in the price sufficient to offset the high (low) real interest rate. Notice further that the higher the speed of adjustment θ, the less will p_c react. It is a slow speed of adjustment in manufactured goods markets (π, to which θ is directly related) that adds to overshooting in the commodity markets.

What determines the long-run equilibrium path $\bar{p}_c$? In the long run, relative prices are determined by exogenous real factors, so

$$(17) \quad \bar{p}_c = \bar{p}_m = \bar{p} = \bar{m} - \phi\,\bar{y} + \lambda(\bar{r} + \mu)$$

where we have used the long-run money demand equation (8). Substituting into (16),

$$(18) \quad p_c = \bar{m} - \phi\bar{y} + \lambda(\bar{r} + \mu)$$
$$- \frac{1}{\theta}(i - \mu - \bar{r}).$$

We see that, in addition to the effect of the real interest rate just discussed, an unanticipated increase in the expected long-run rate of money growth μ increases the current $\bar{p}_c$ and therefore the current p_c. We thus have what we wanted, a model of commodity prices that shows both the negative effect of the real interest rate and the positive effect of the expected long-run money growth rate.

To quantify the immediate impact of a change in the level or growth rate of the money supply, we take the change in equation (1), then use equation (15) to get the change in the rationally expected rate of appreciation and use equation (17) to get the change in $\bar{p}_c$:

$$(19) \quad \Delta i = \Delta \dot{p}_c^e$$
$$= -\theta\Delta(p_c - \bar{p}_c) + \Delta\mu$$
$$= -\theta\Delta p_c + \theta\Delta m + (1 + \theta\lambda)\Delta\mu.$$

Finally, we take the change in equation (5), keeping in mind that only i and p_c, not p_m, are free to respond to a monetary disturbance,

$$(1 - \alpha)\Delta p_c - \Delta m = \lambda\Delta i;$$

we combine with equation (19) to obtain our result:

$$(20) \quad \Delta p_c = \frac{1 + \lambda\theta}{1 - \alpha + \lambda\theta}\Delta m$$
$$+ \lambda\frac{1 + \lambda\theta}{1 - \alpha + \lambda\theta}\Delta\mu.$$

We can see that when there is a change in the level of the money supply, p_c initially overshoots its long-run equilibrium because the coefficient of Δm is greater than unity. In the limit as θ goes to infinity, the coefficient goes to unity and there is no overshooting.

A change in the rate of growth μ changes the long-run equilibrium price by $\lambda\Delta\mu$. The degree of initial overshooting is then the same as for the change in the level.

To summarize, monetary policy has an effect on real agricultural commodity prices even though they are flexible, because the prices of other goods are sticky. A decline in the level of the money supply in the short run raises the real interest rate, which depresses commodity prices. Commodity prices will fall more than proportionately to the change in the money supply; they overshoot their new long-run equilibrium. For commodities to be willingly held, they must be sufficiently undervalued that there is an expectation of future price increases large enough to offset the higher real interest rate. This expectation will subsequently turn out to be rational, as the general price level rises over time, and the reduction in the real money supply and its effects on the real interest rate and real commodity prices disappear over time. A decline in the rate of growth of the money supply has similar overshooting effects on commodity prices. An implication is that macroeconomic policy may be as important a source of fluctuations in agricultural prices as the traditional microeconomic factors.

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